Single-pair measurement of the Bell parameter: certifying entanglement without destroying it

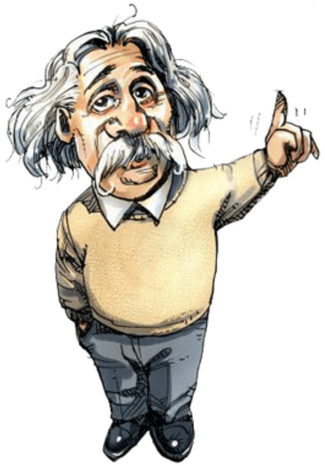
F. Piacentini, S. Virzì, E. Rebufello, F. Atzori, A. Avella, R. Lussana, I. Cusini, F. Madonini, F. Villa, M. Gramegna, E. Cohen, I. P. Degiovanni, and M. Genovese

19NET02 EMN Quantum

19NMR06 MeTISQ

PATHOS FET-OPEN

The EPR paradox



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

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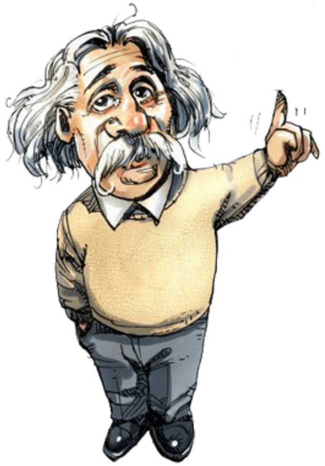

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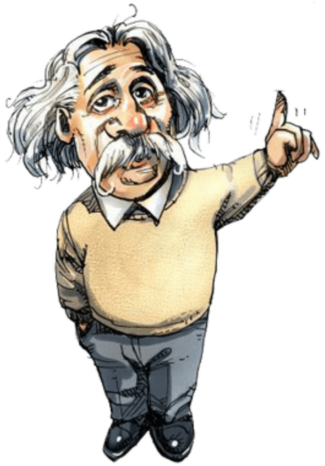

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What if the Locality assumption is added? → *Local* Hidden Variable Theories (LHVTs)

Bell Inequalities

Quantum Mechanics

VS

LHVTs

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- Non-epistemic probabilistic description of the laws of nature;

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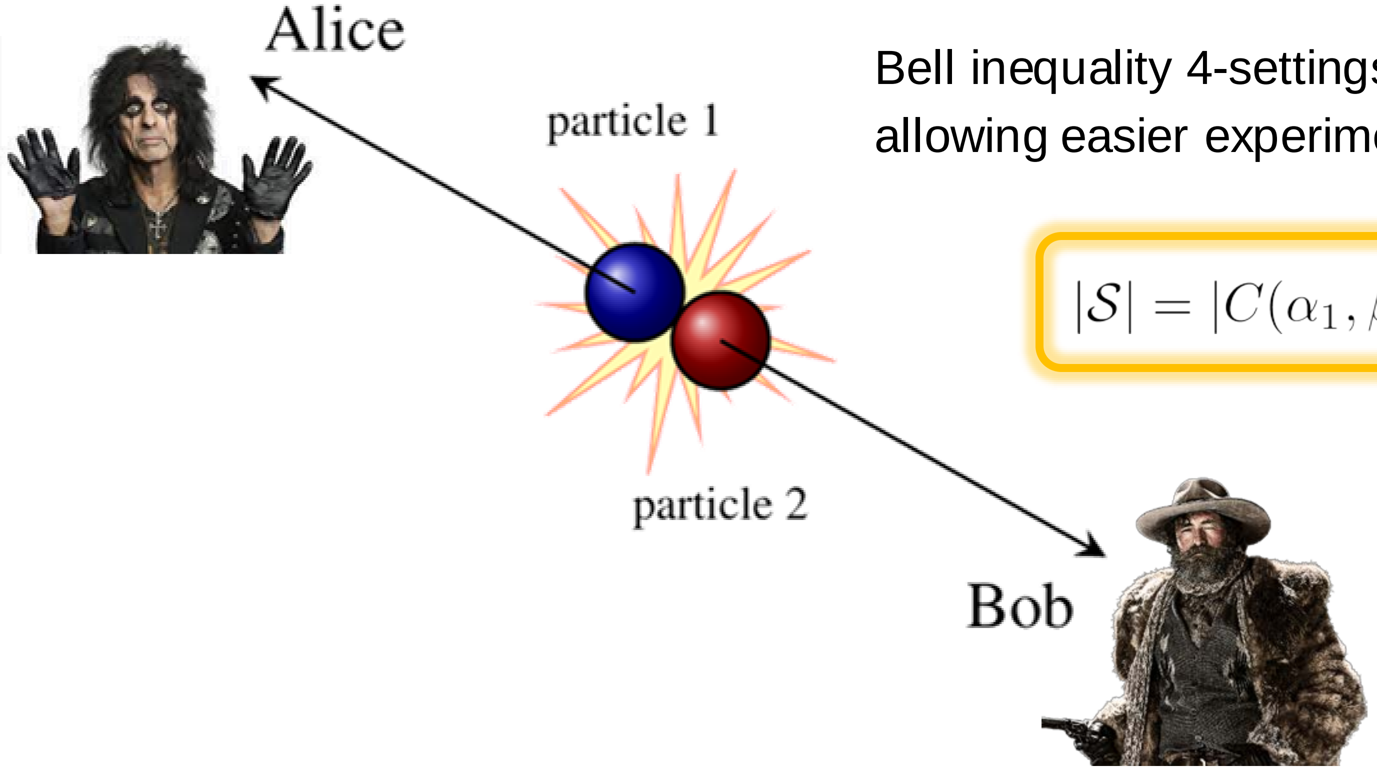
The CHSH inequality



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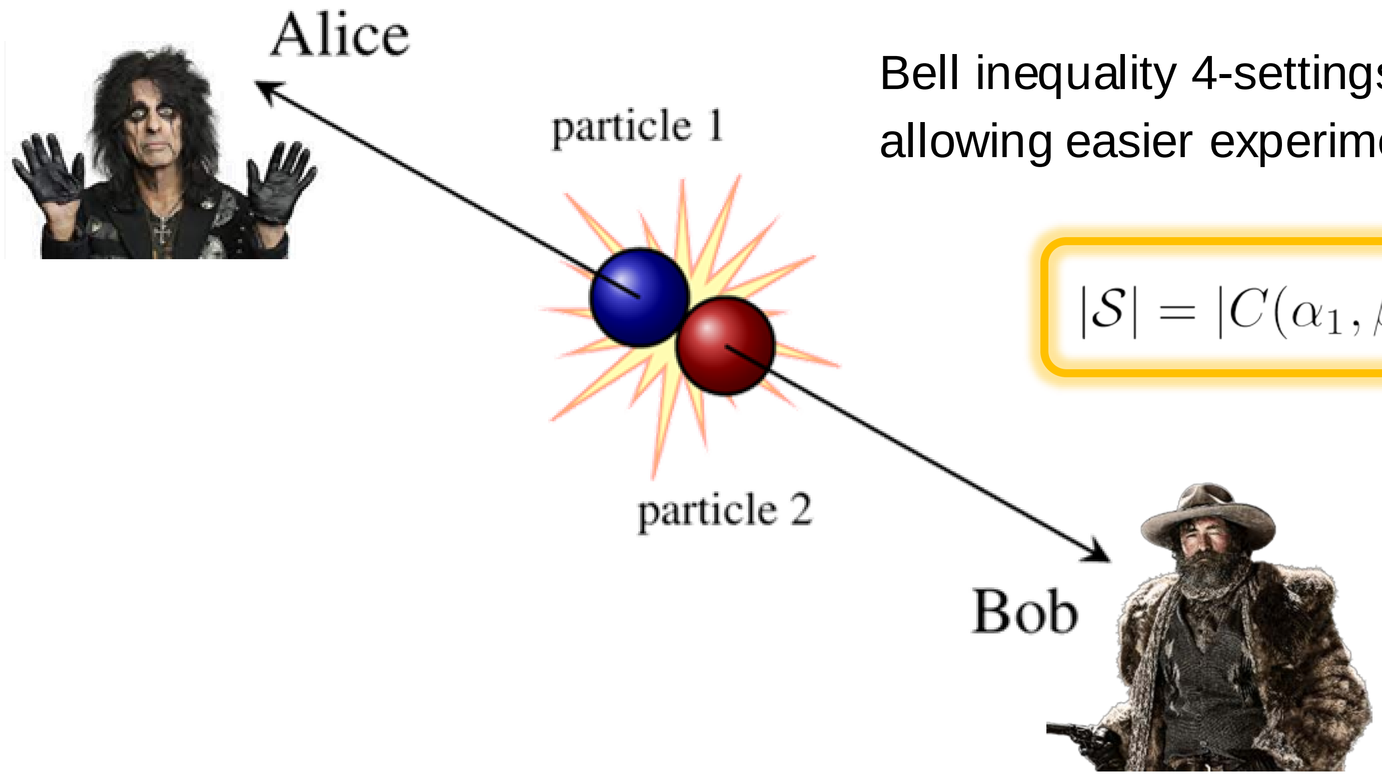

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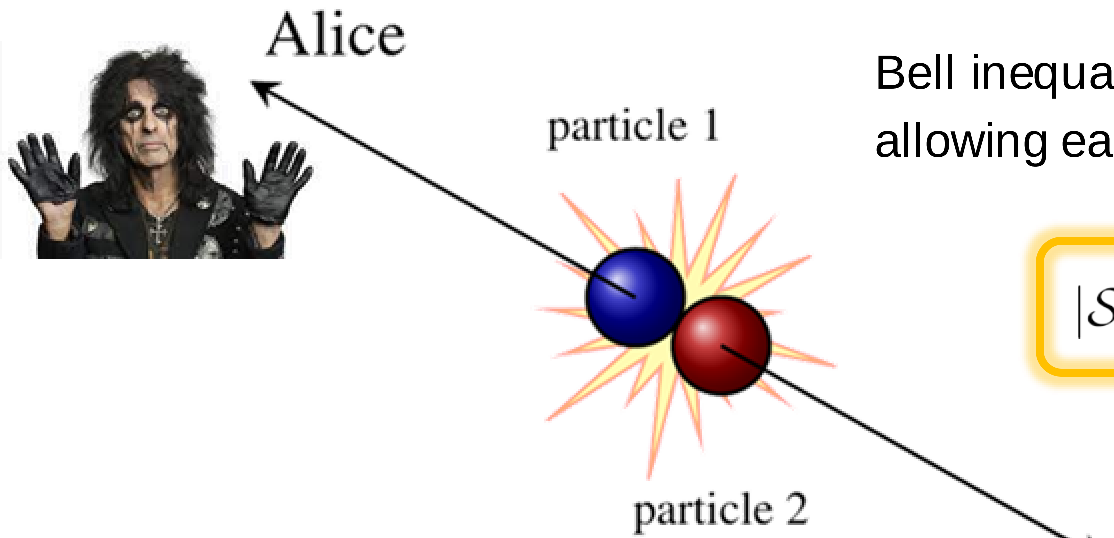

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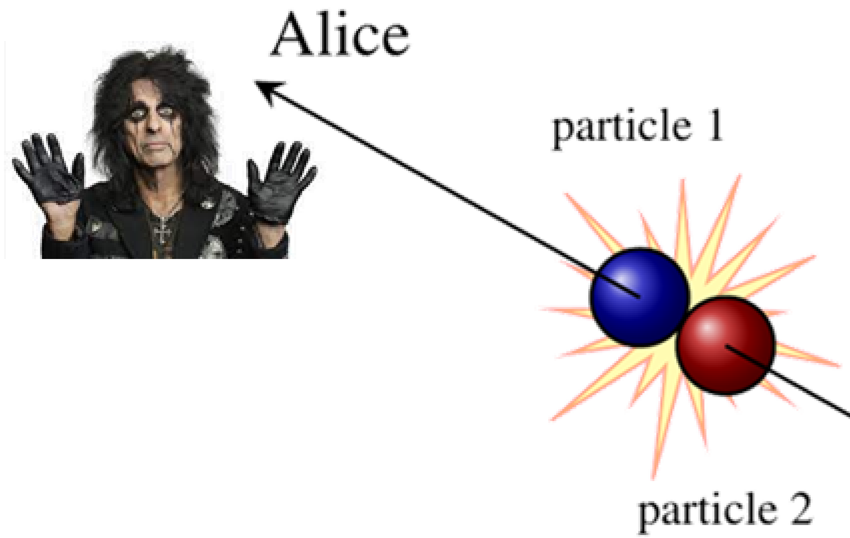

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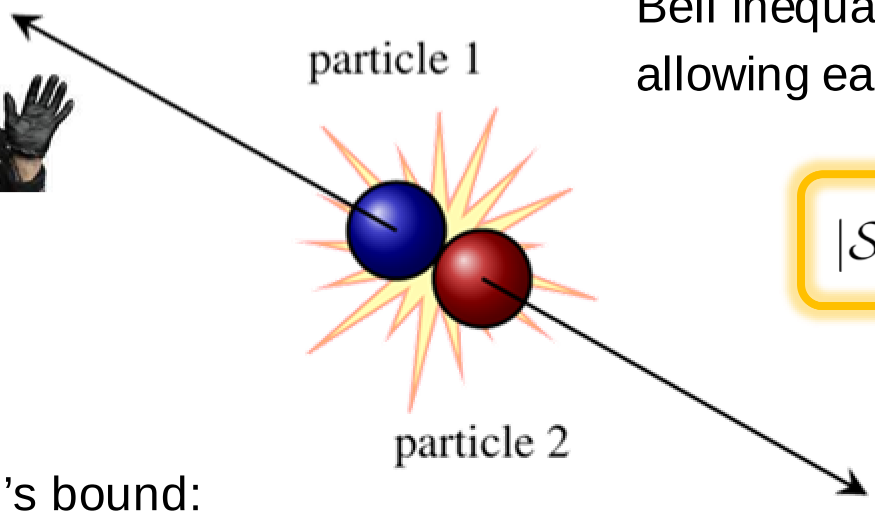

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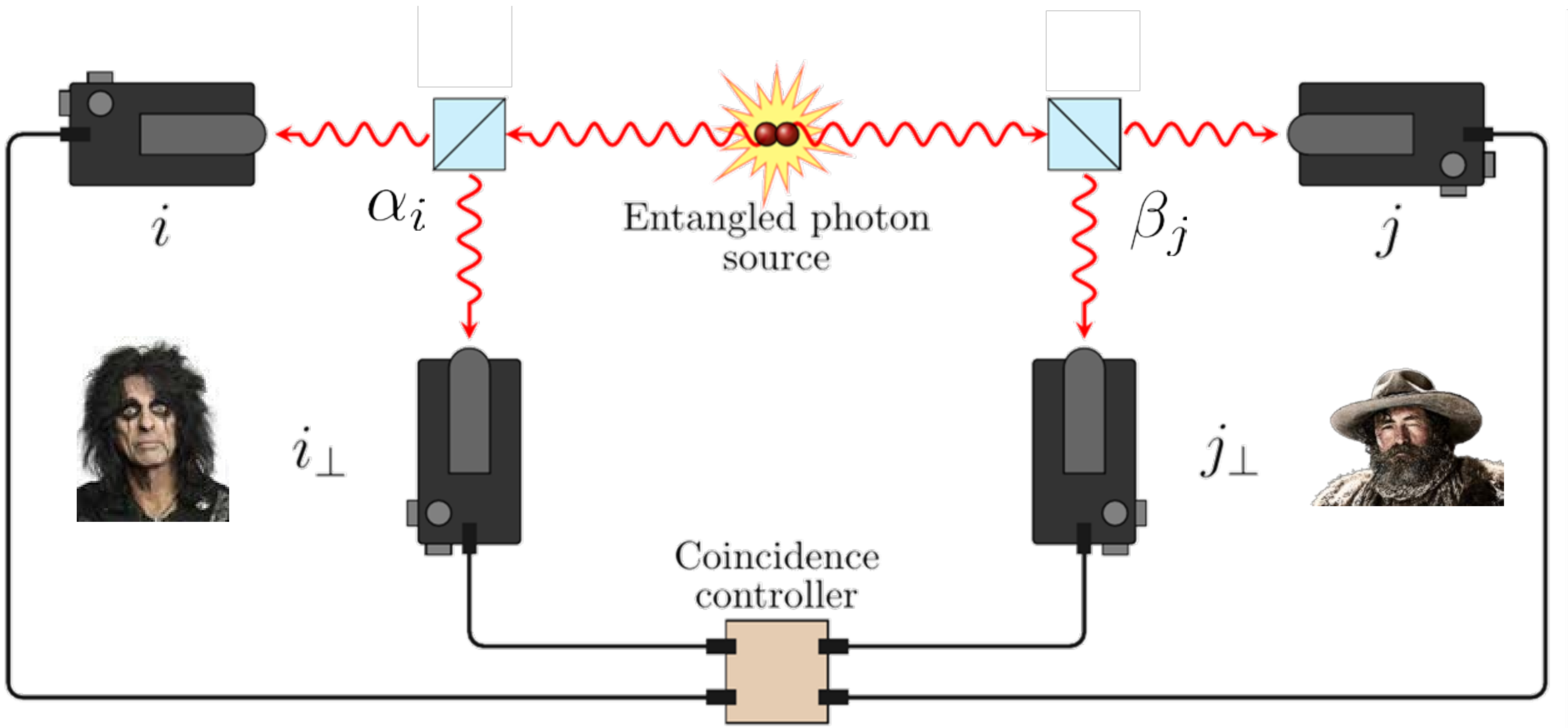

To estimate S, Alice and Bob have to randomly choose their measurement settings in each experimental run.

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Weak Measurements [Aharonov, Albert & Vaidman, PRL 60 (1988)] - little information is extracted from a single measurement event, but the state does NOT collapse: incompatible measurements on the same quantum state are allowed!

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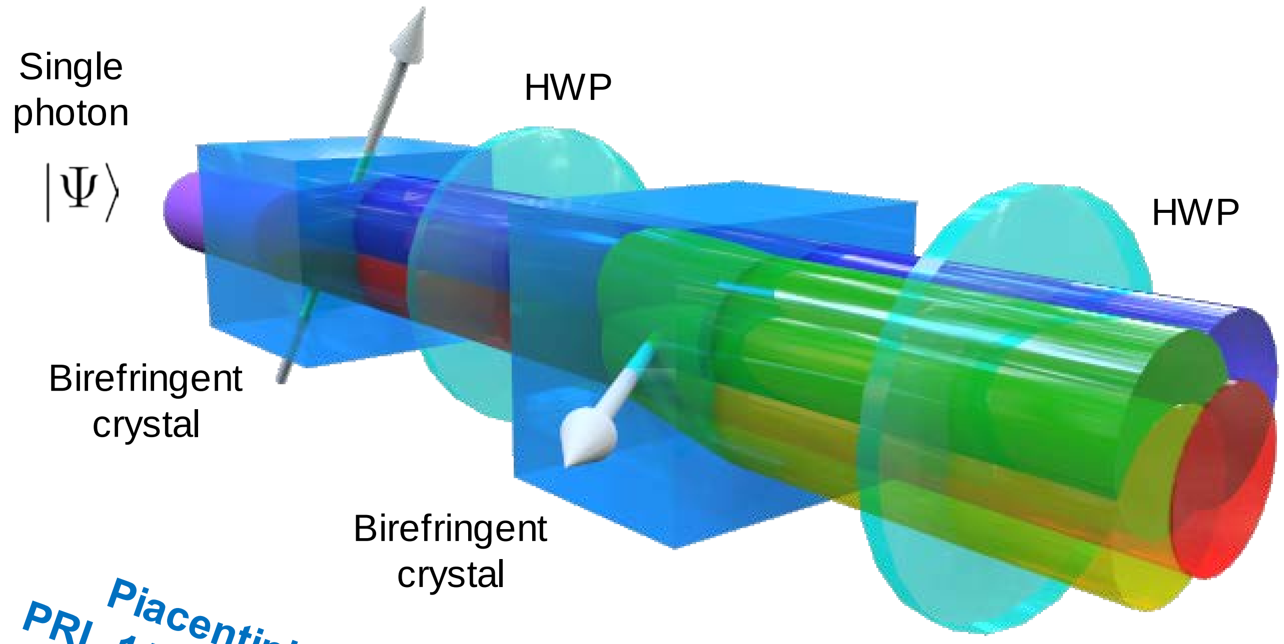

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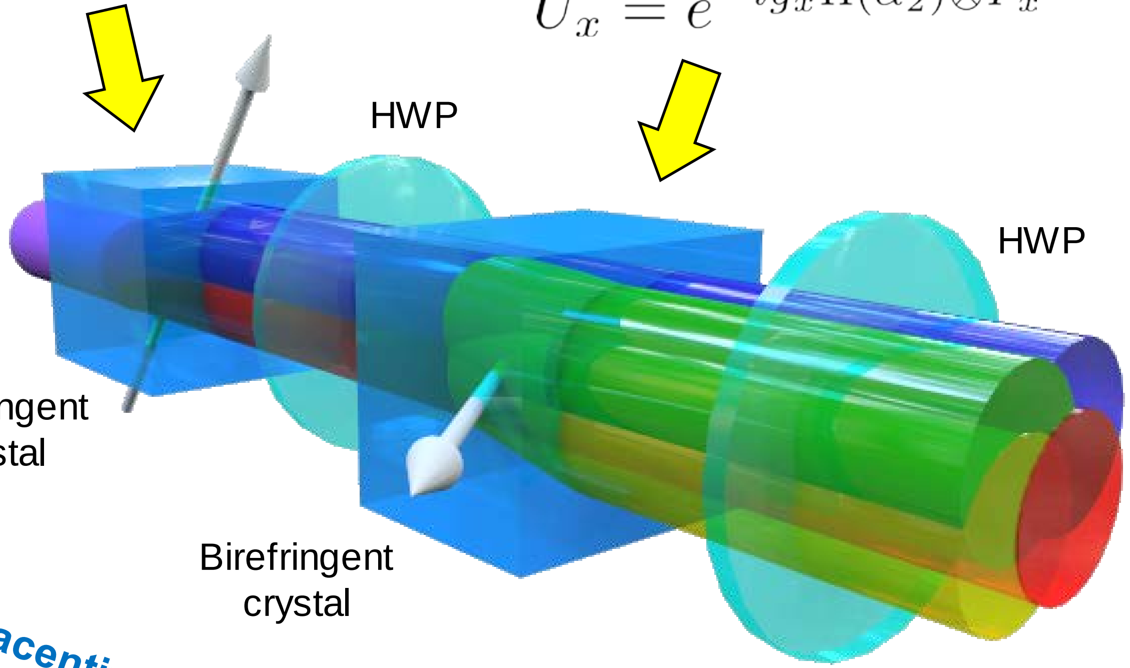

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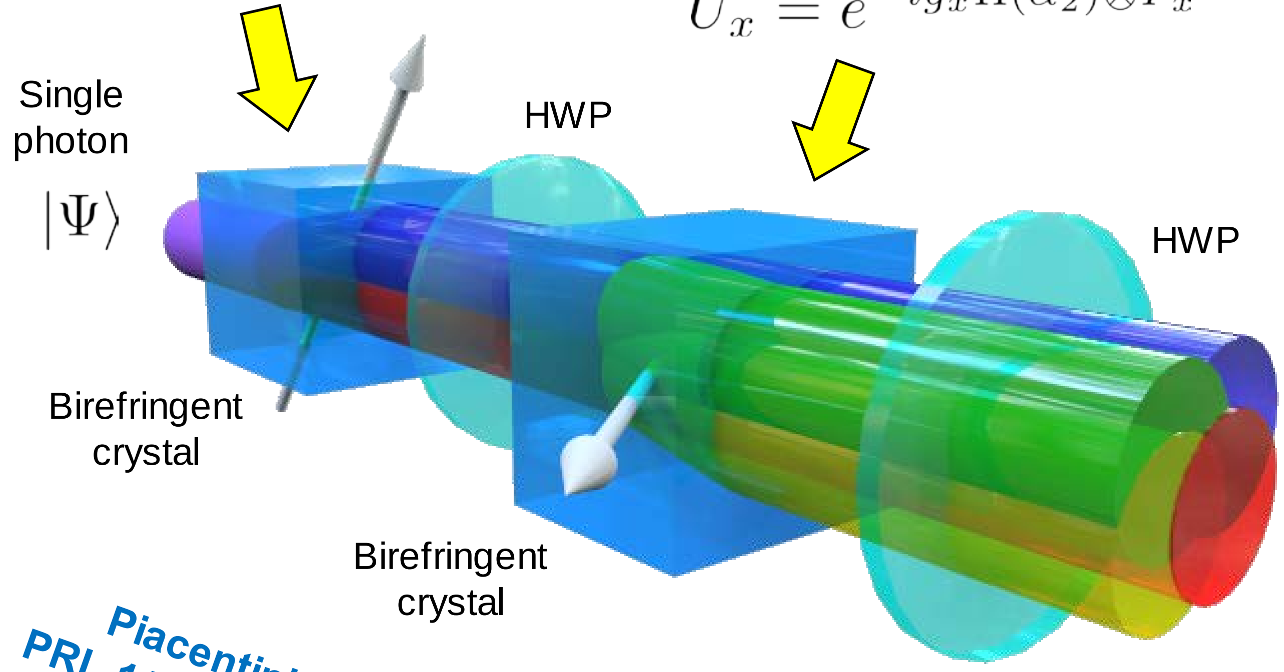

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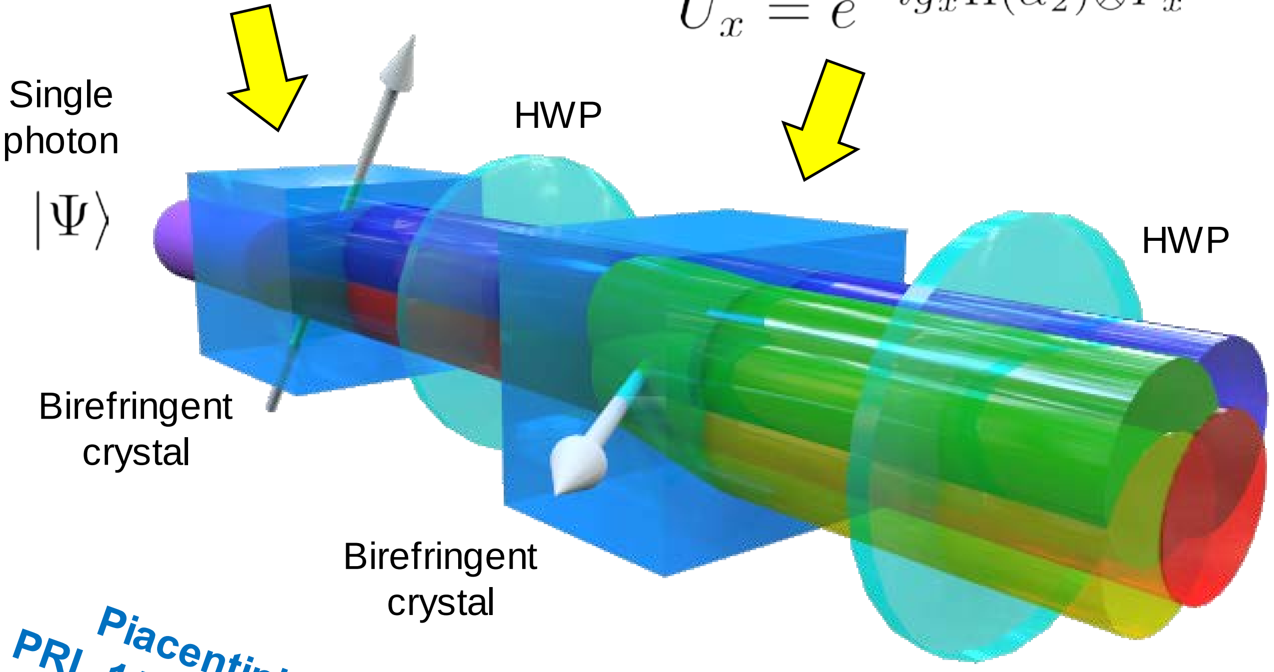

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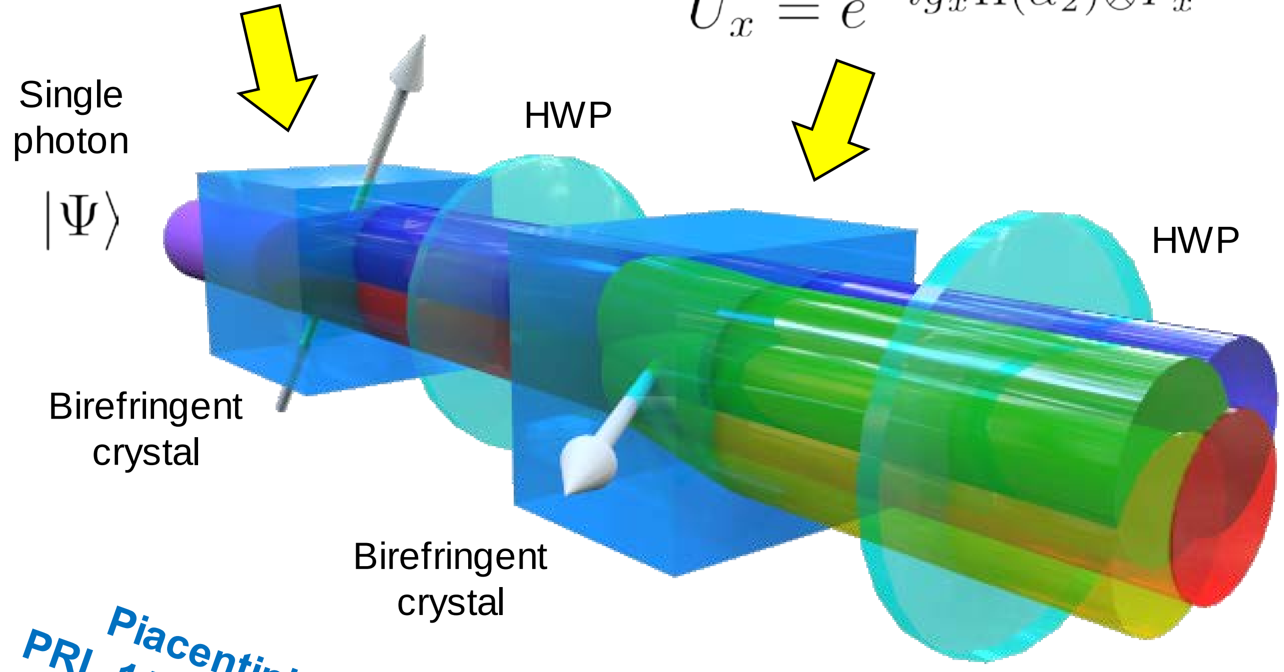

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Next step: from single photons to entangled pairs

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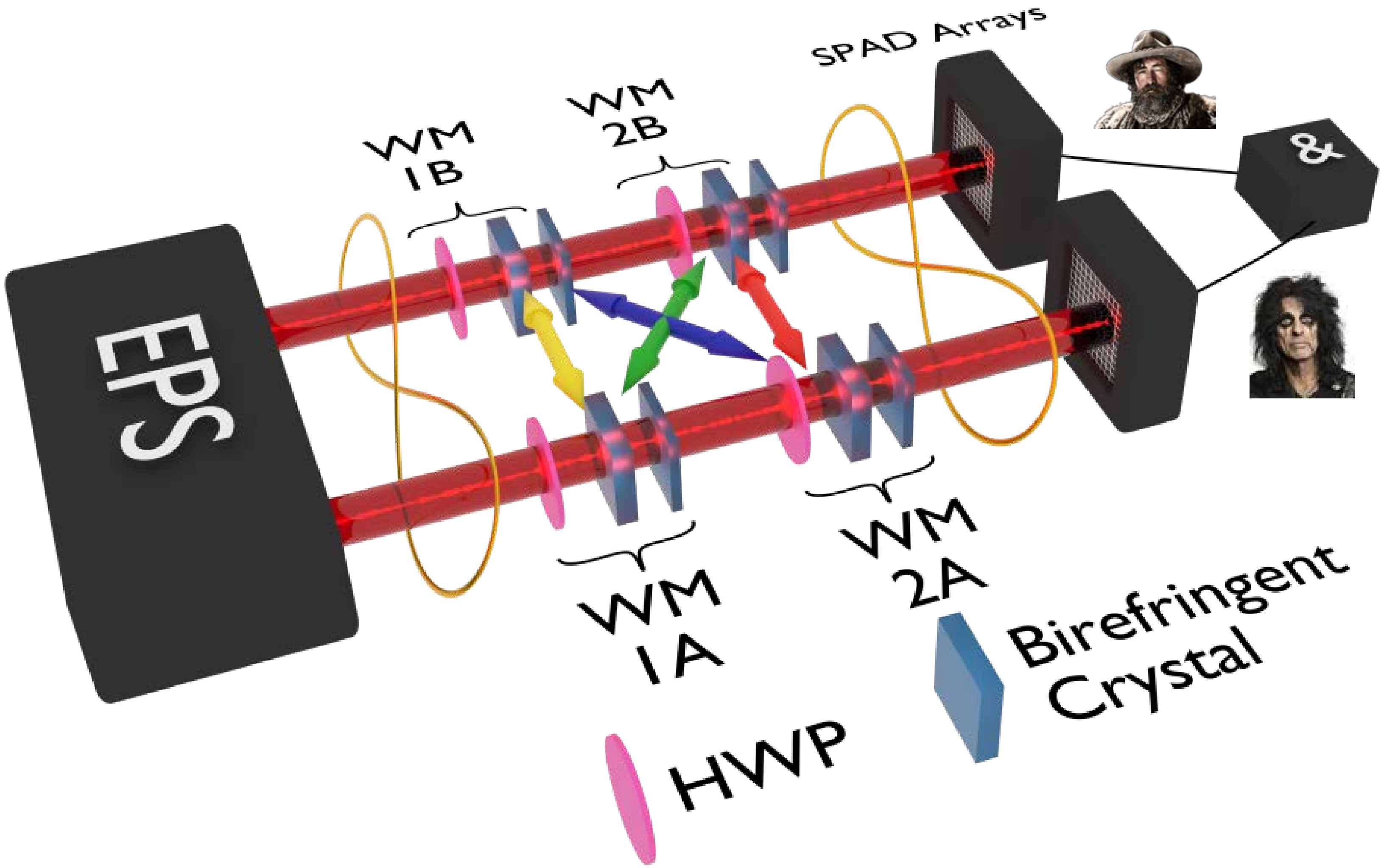

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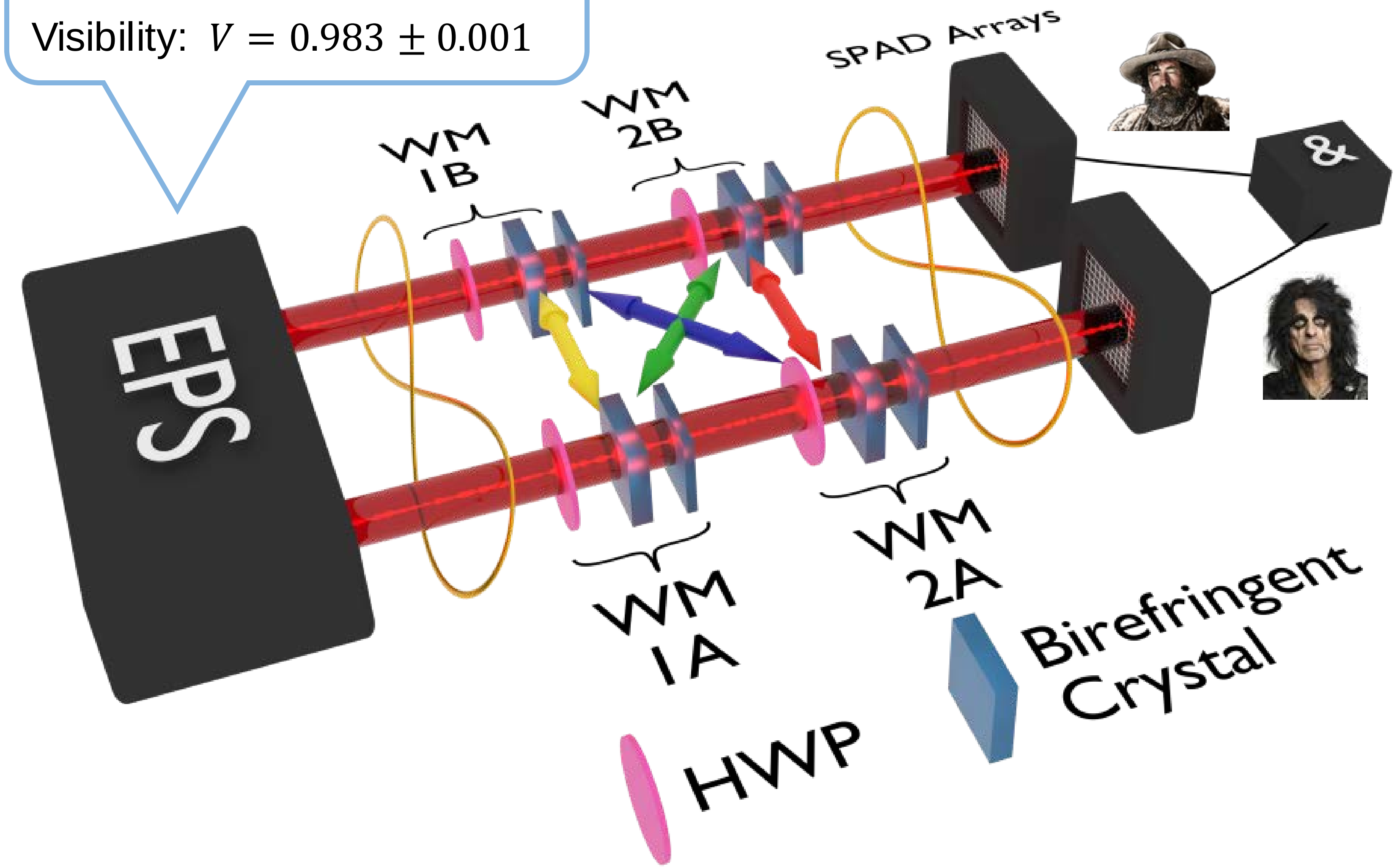
Visibility: $V = 0.983 \pm 0.001$



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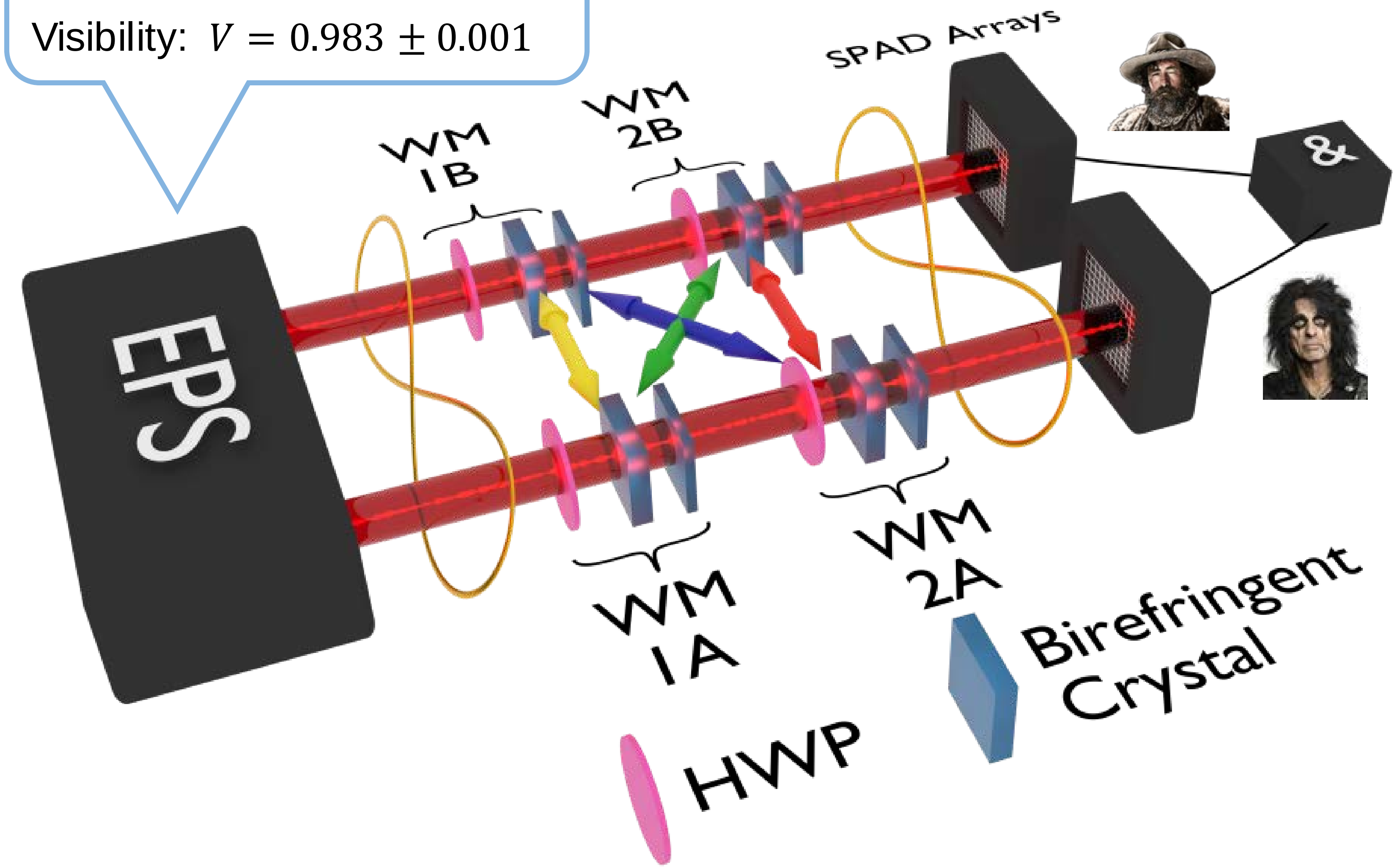

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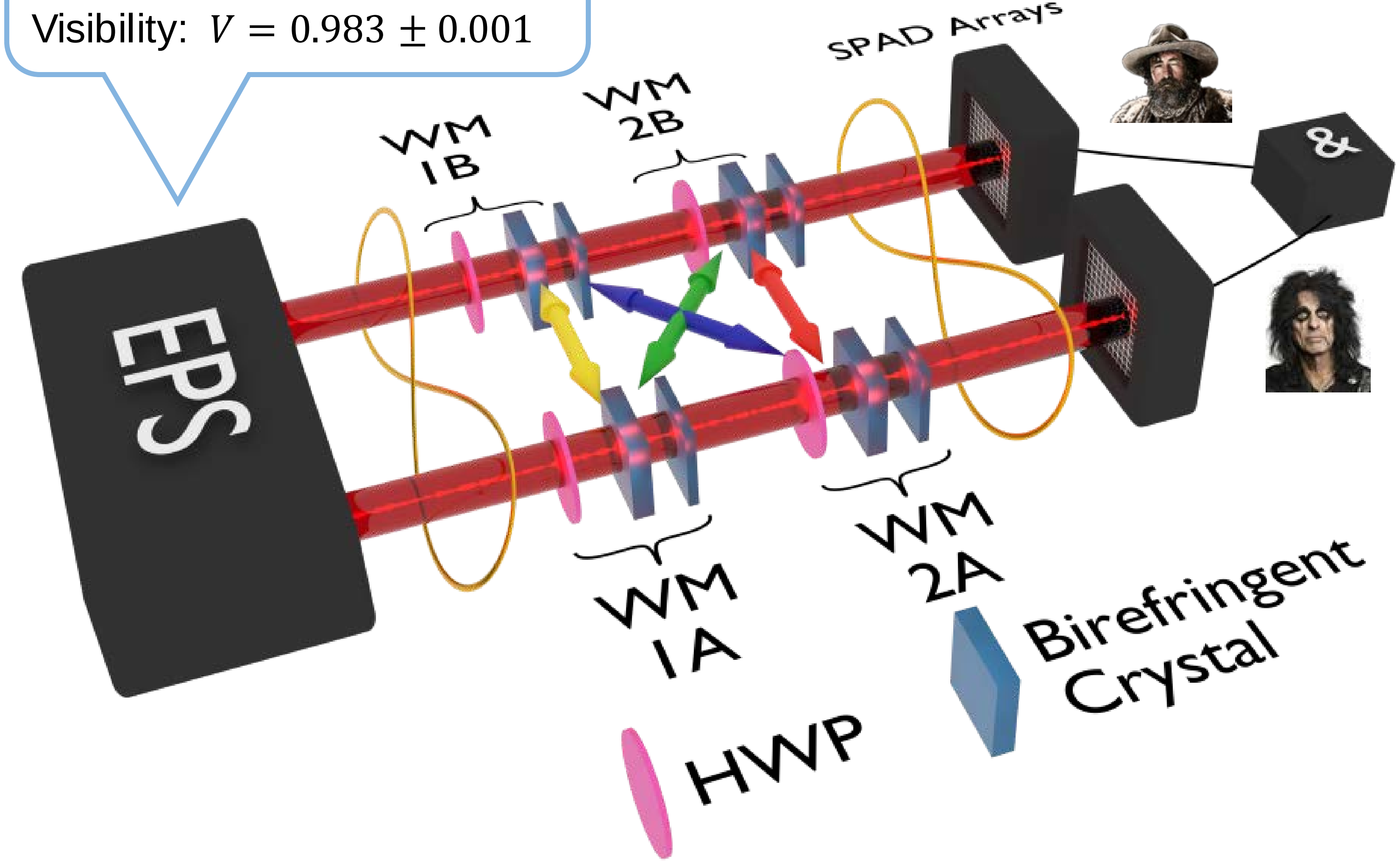

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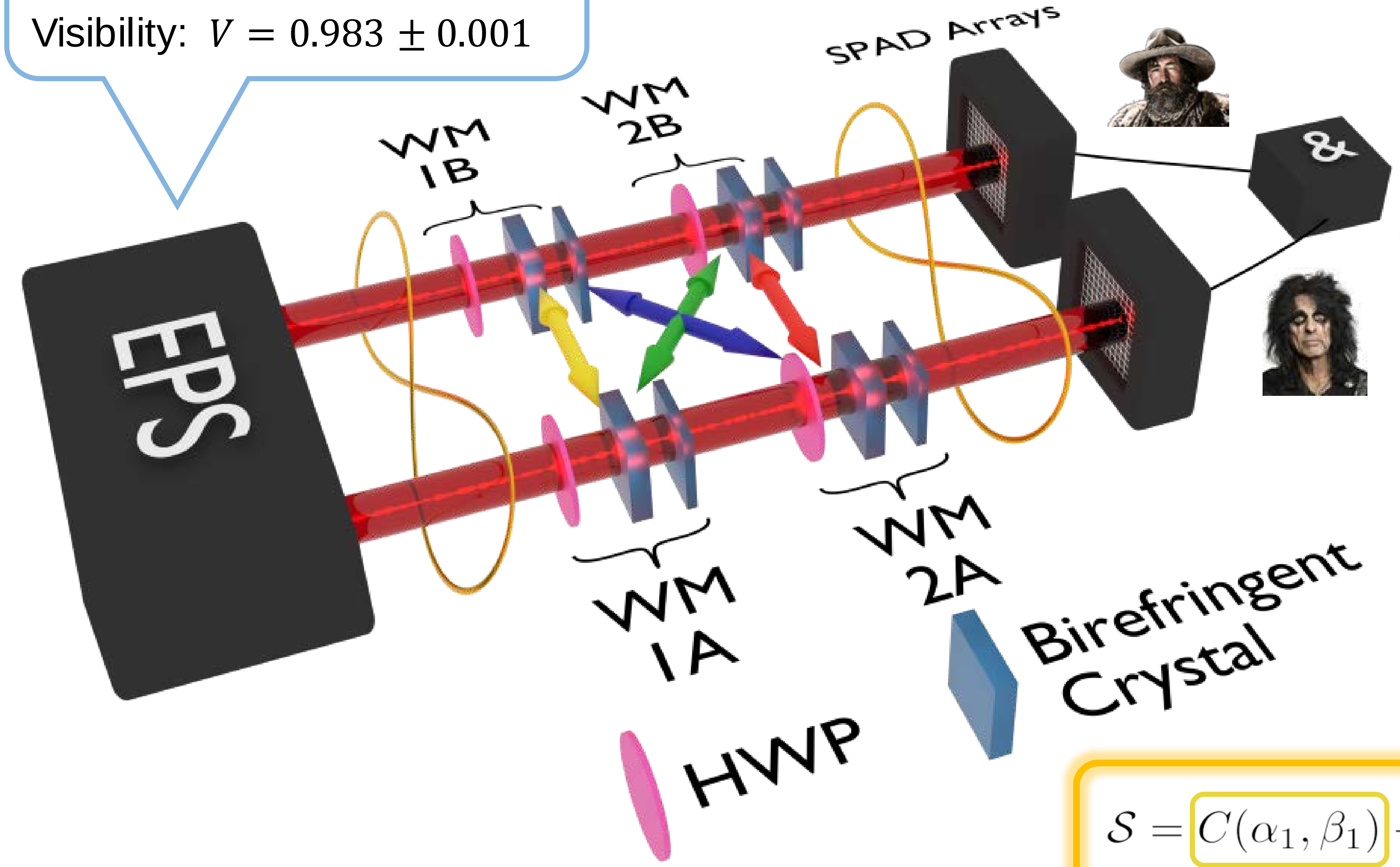

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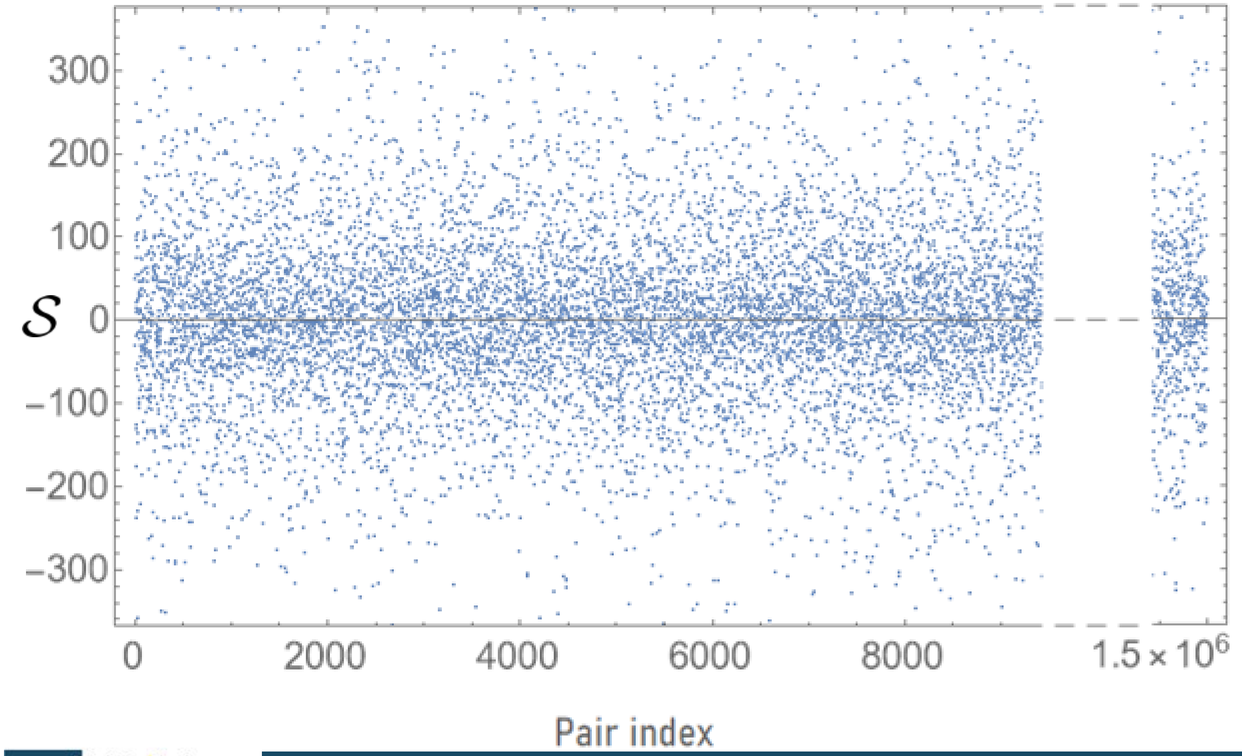

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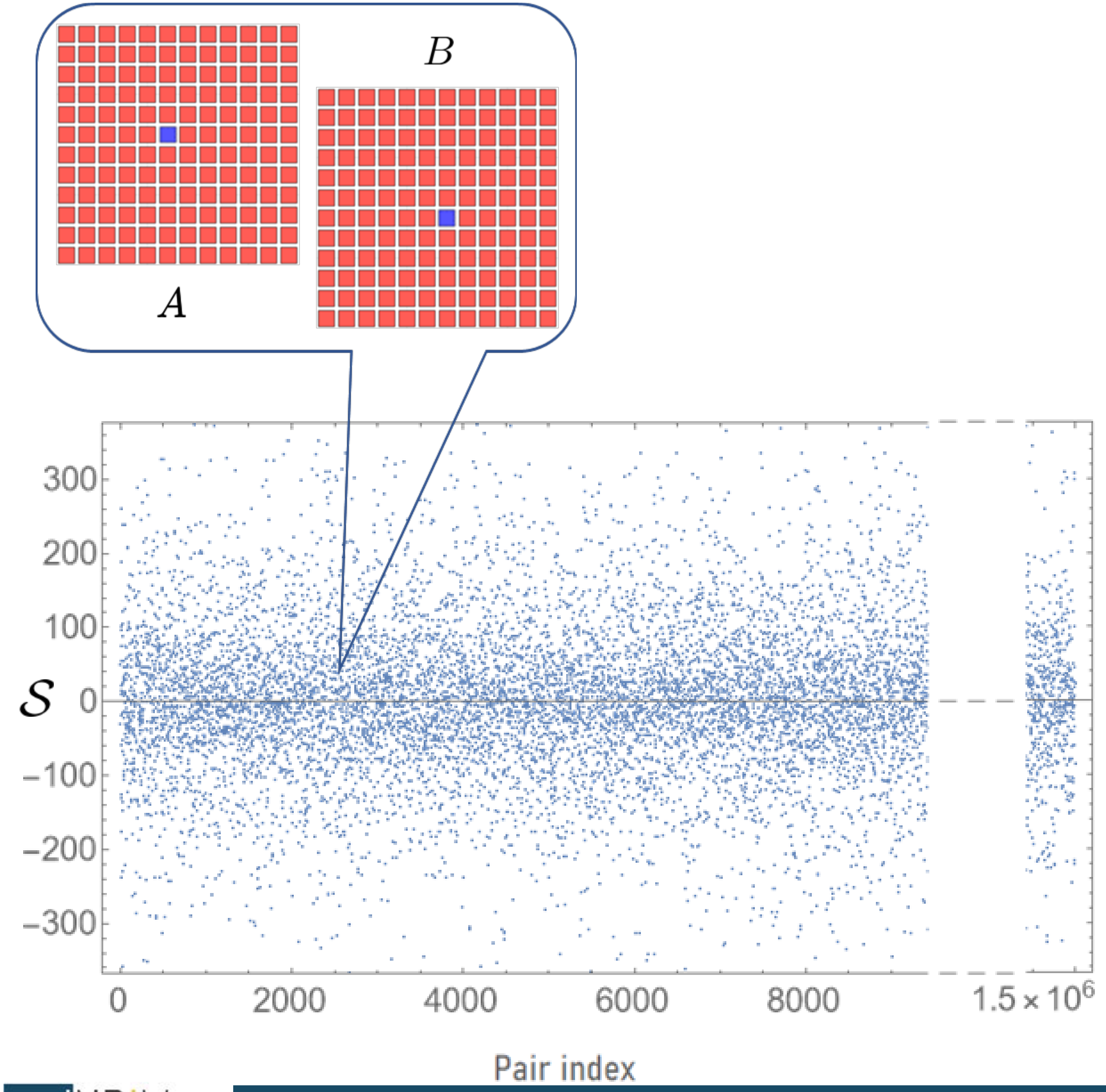

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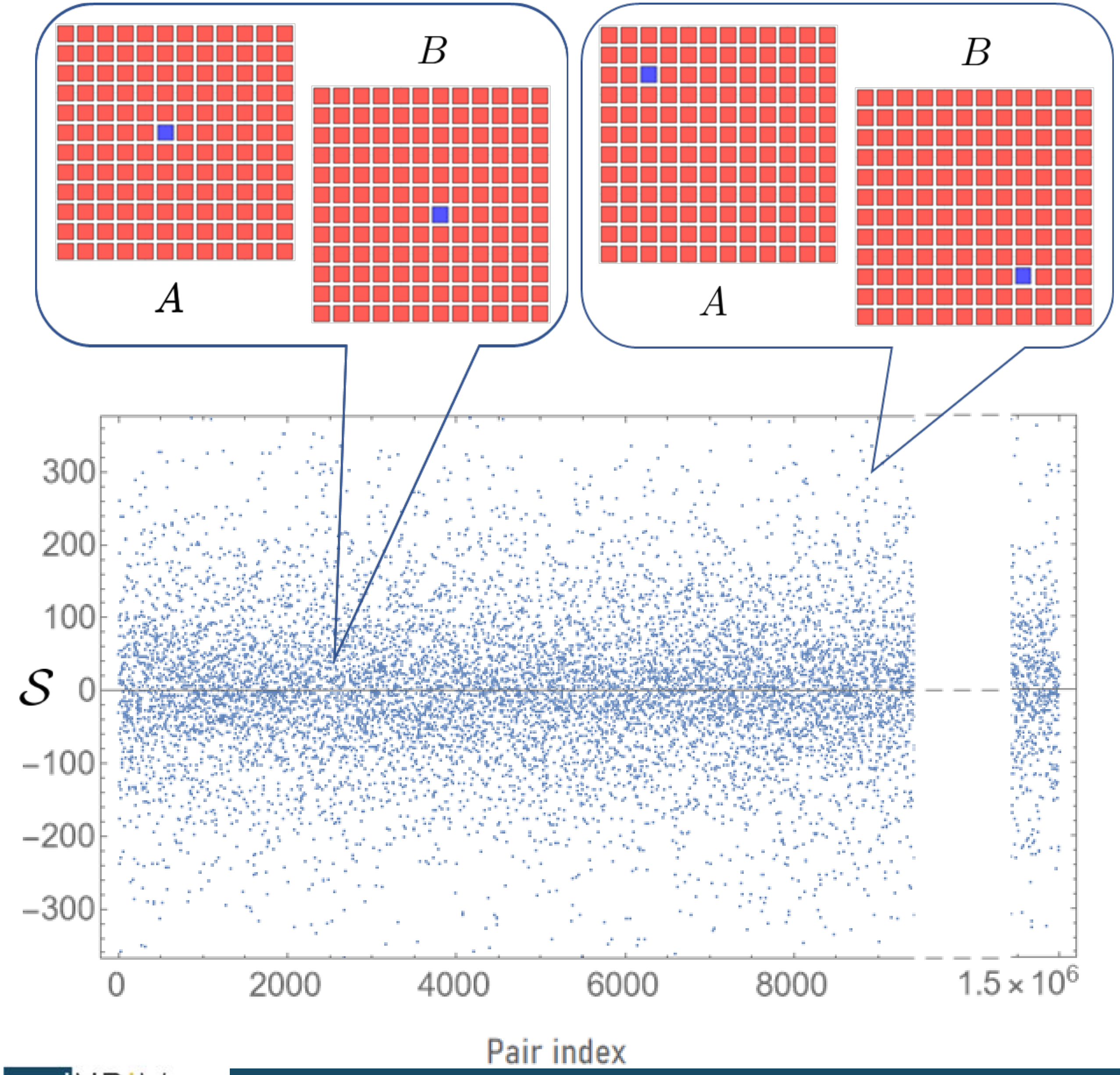

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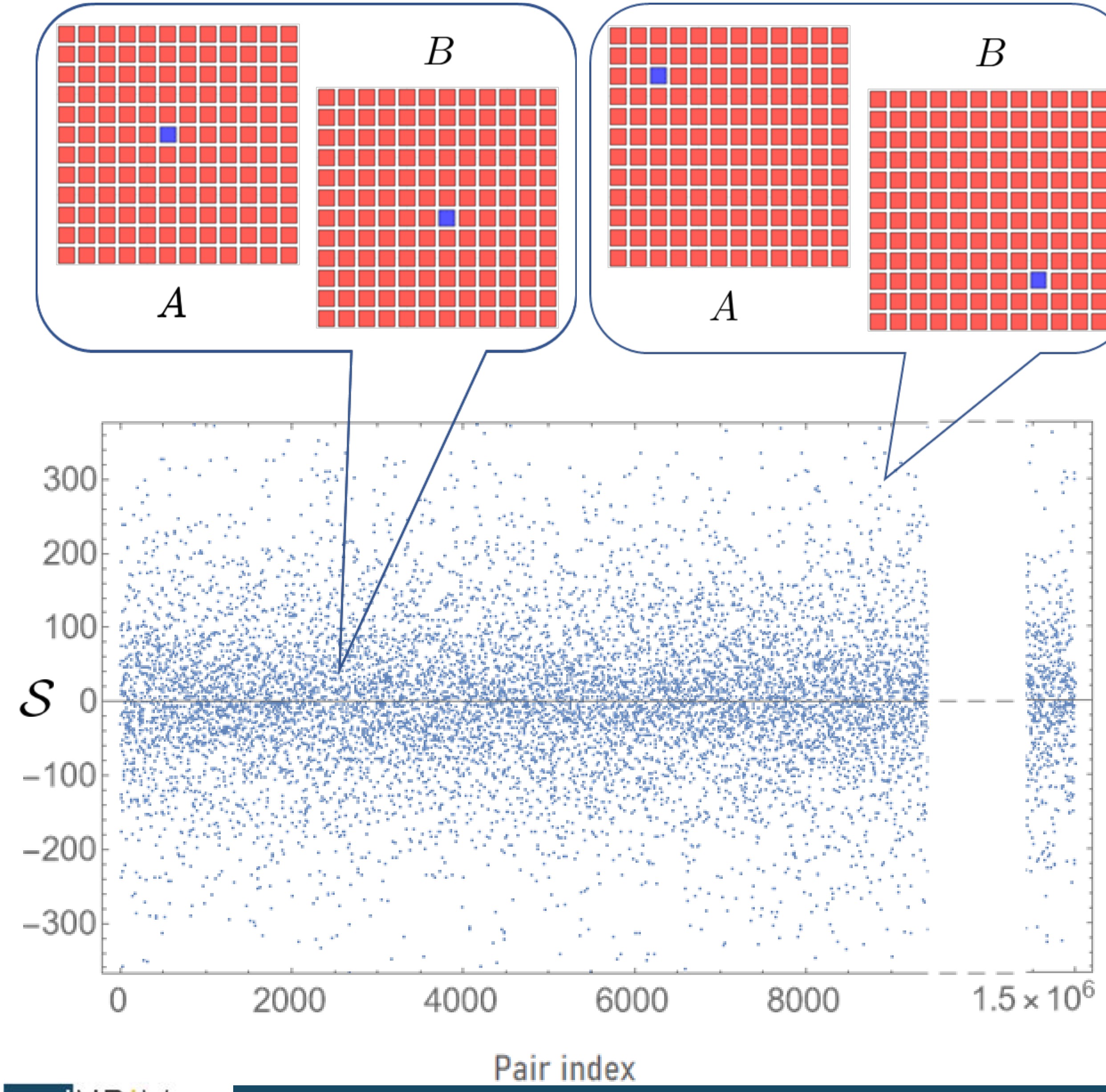

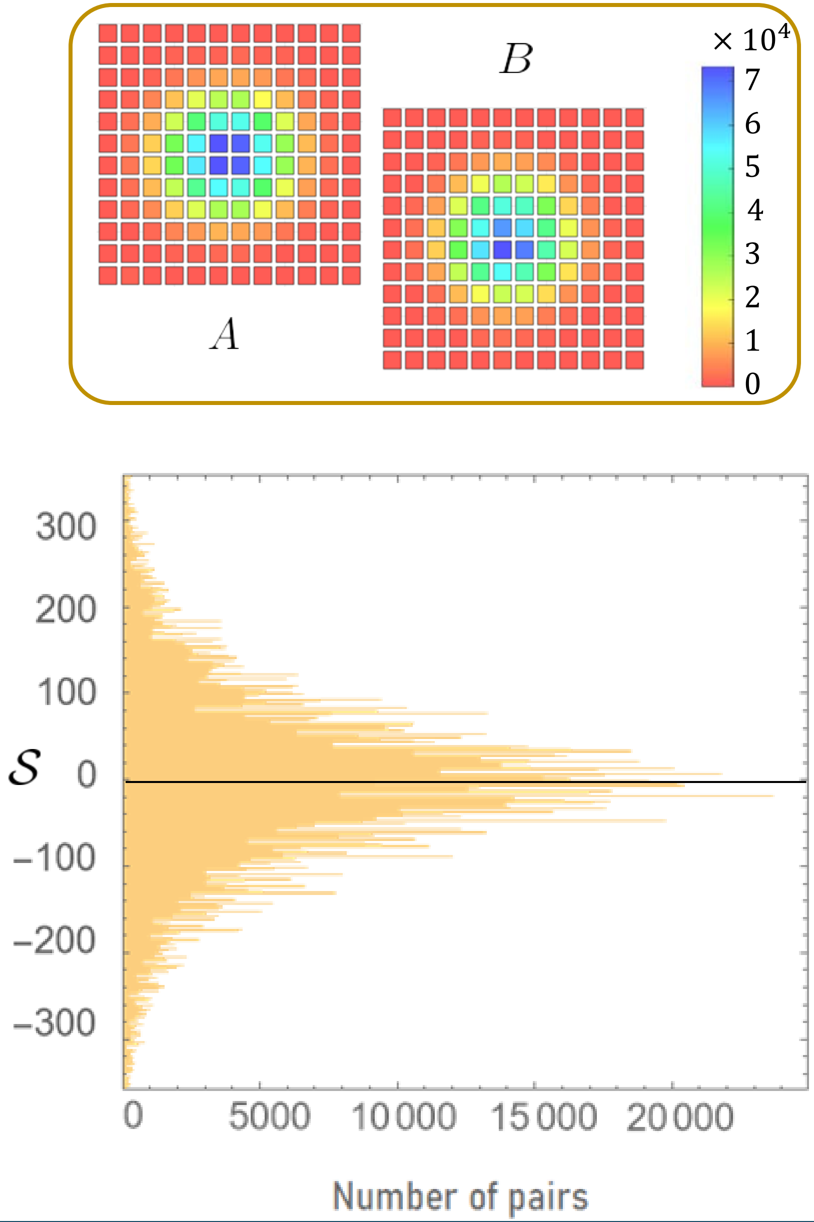

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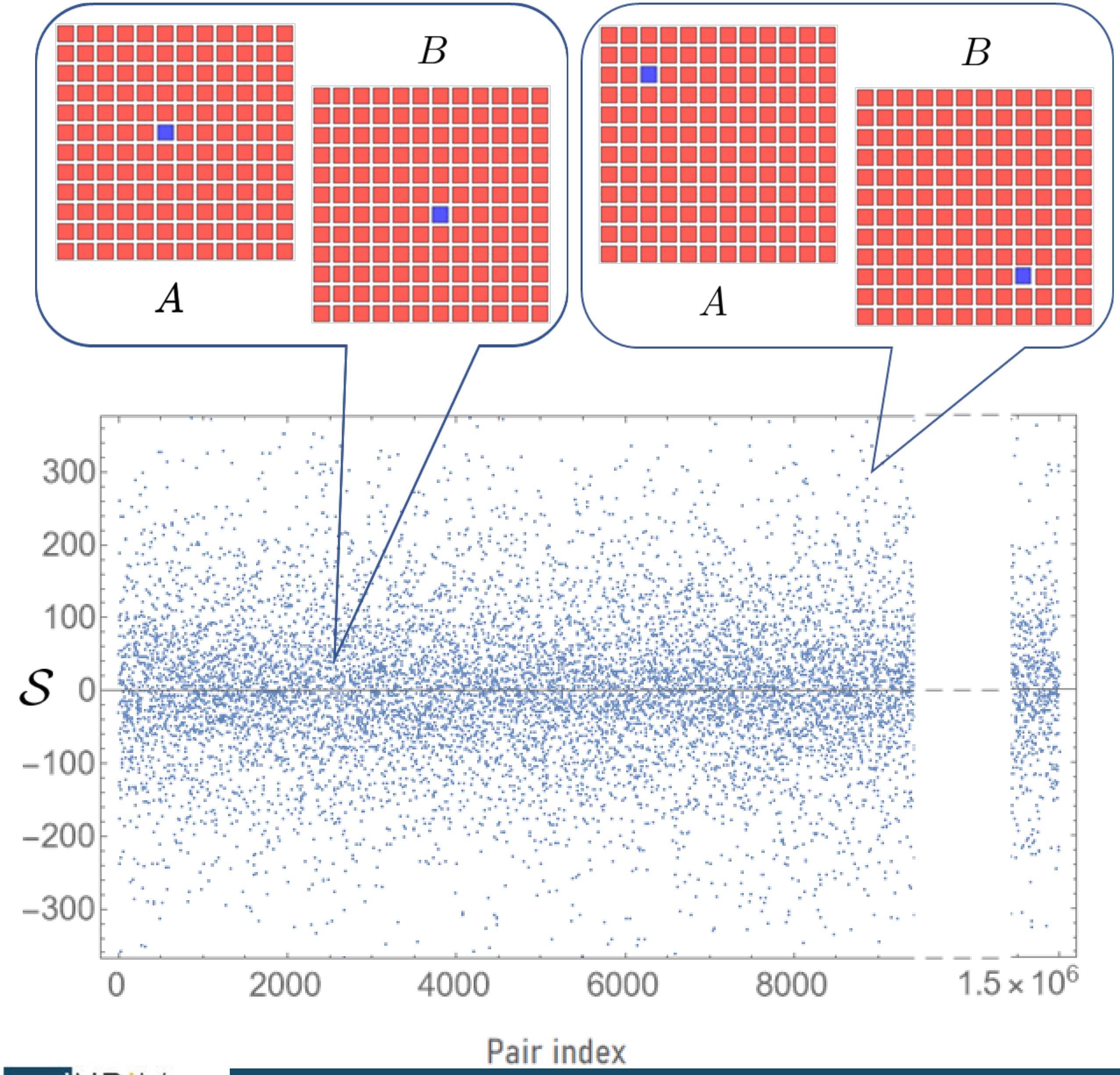
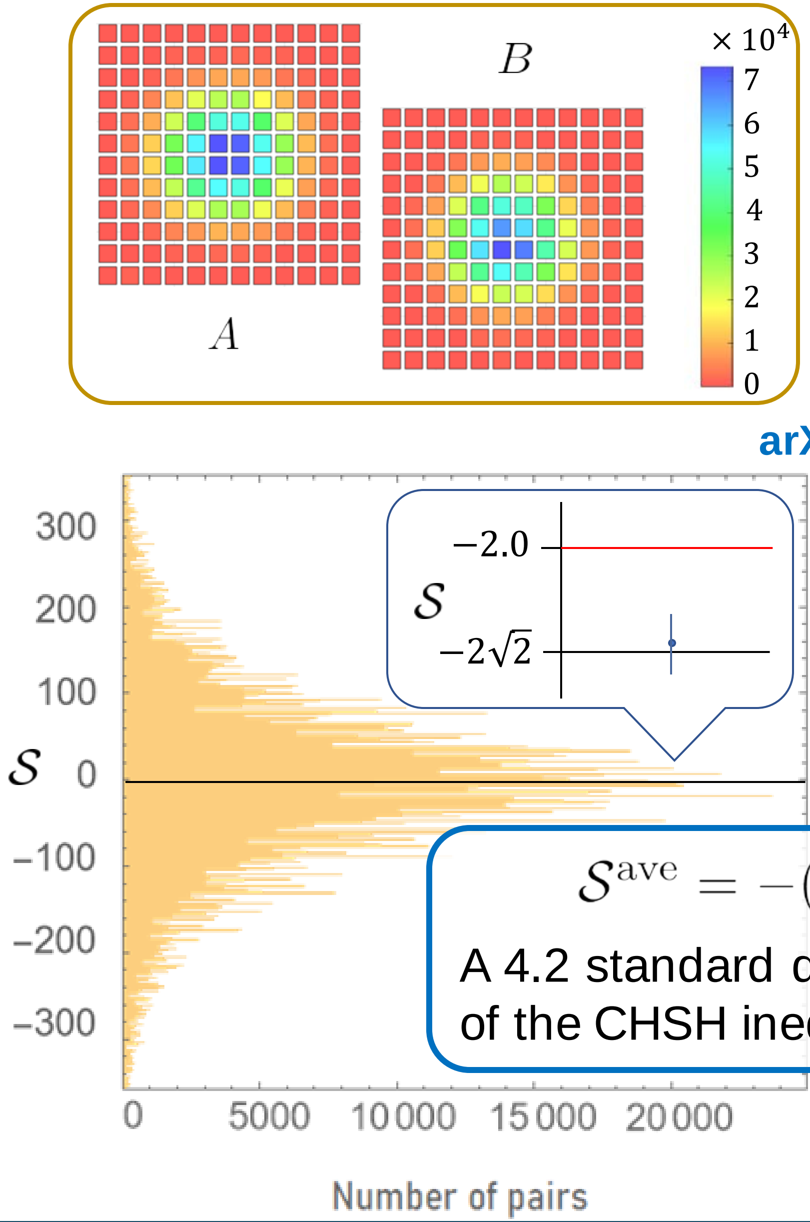




Virzì et al., arXiv:2303.04787 (2023)

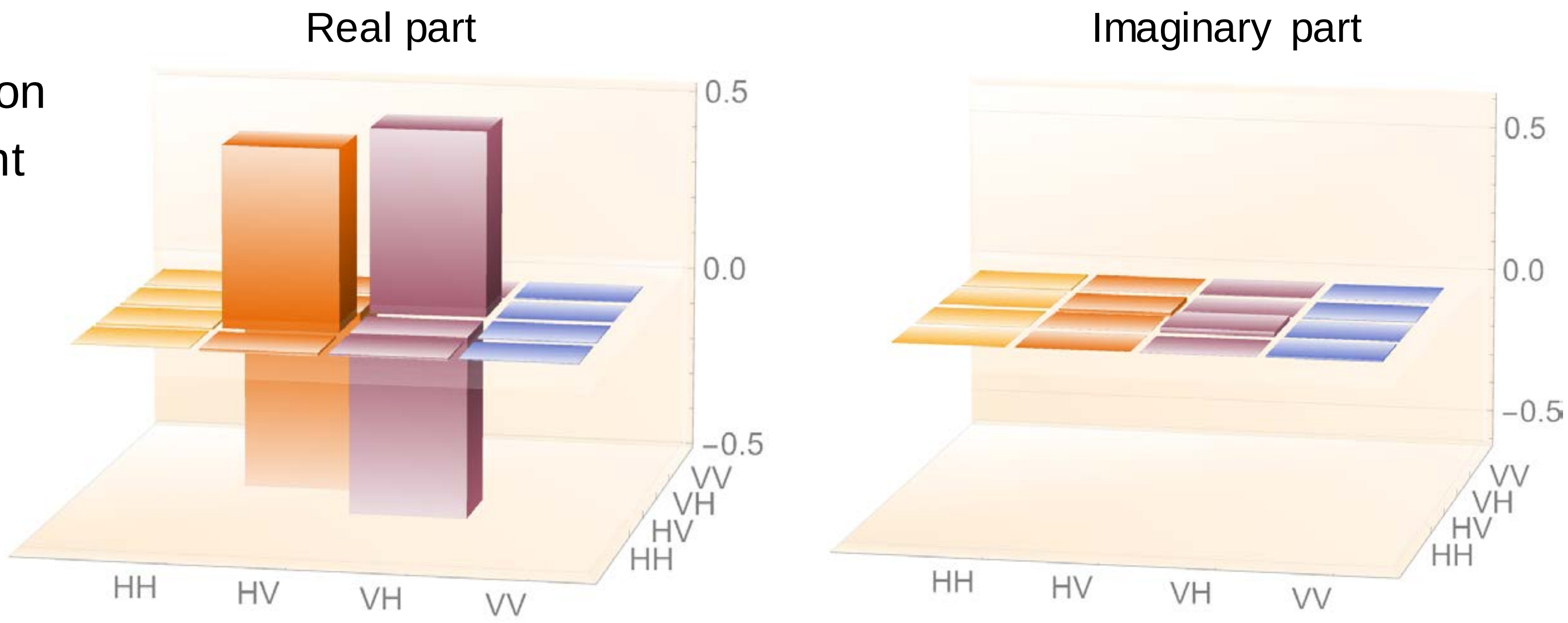
$$\mathcal{S}^{\mathrm{ave}} = -(2.79 \pm 0.19)$$

A 4.2 standard deviations violation of the CHSH inequality is achieved

What about the two-photon state after the measurement?

Tomographic reconstruction of the two-photon state before the Bell parameter measurement

Virzì et al.,
arXiv:2303.04787 (2023)



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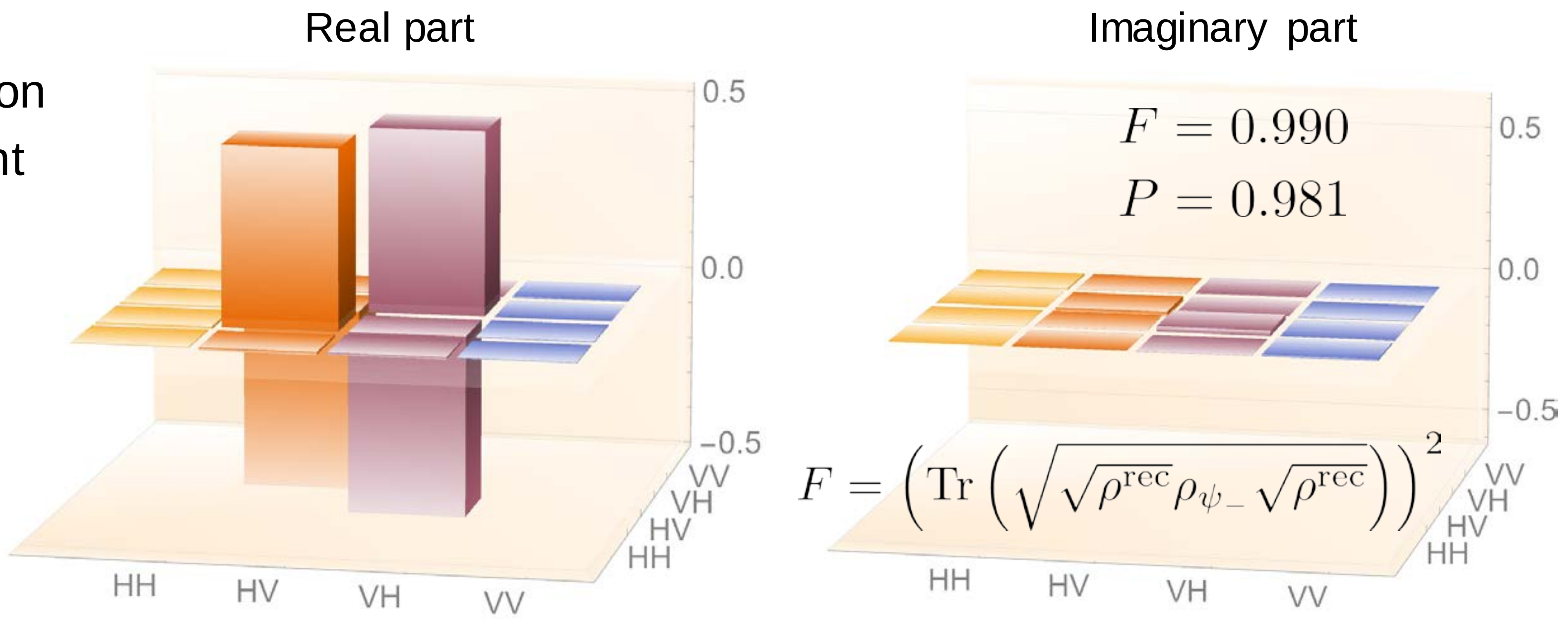

$$F = \left(\mathrm{Tr}\left(\sqrt{\sqrt{\rho^{\mathrm{rec}}}\,\rho_{\psi_-}\sqrt{\rho^{\mathrm{rec}}}}\right)\right)^2$$

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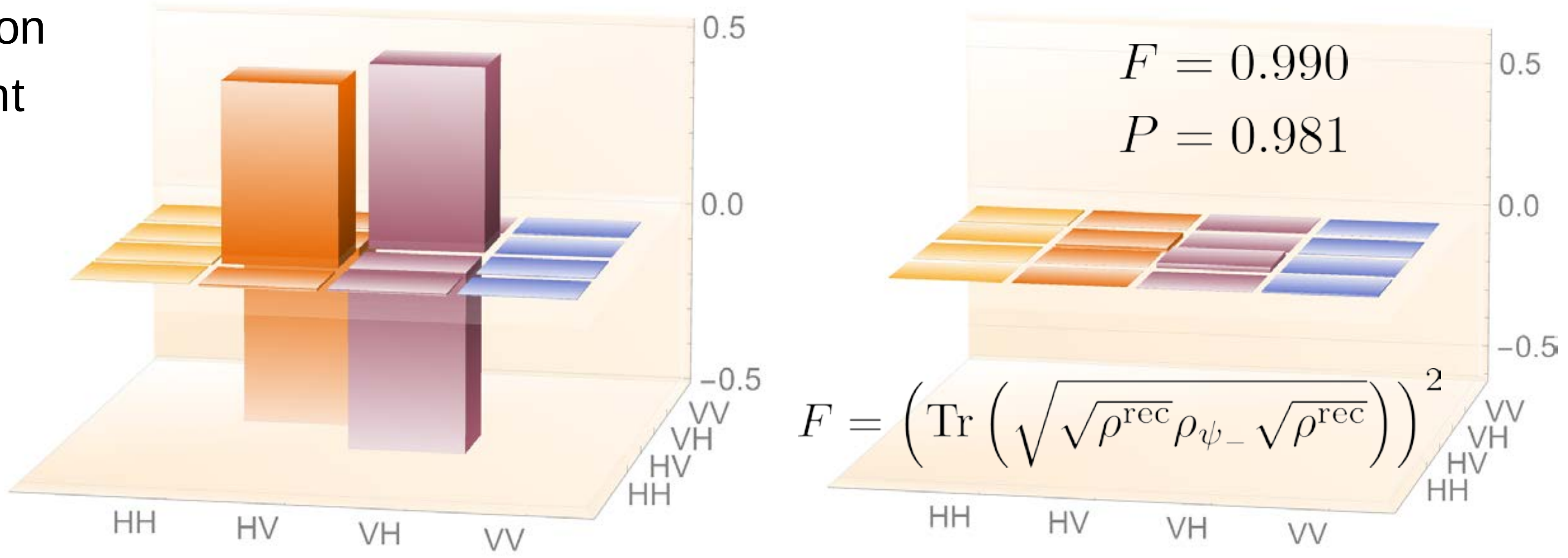
Negativity: $\mathcal{N}^{\text{in}} = 0.981$

Concurrence: $\mathcal{C}^{\text{in}} = 0.979$

Virzì et al., arXiv:2303.04787 (2023)

Real part | Imaginary part



$F = 0.990$

$P = 0.981$

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Tomographic reconstruction of the two-photon state after the Bell parameter measurement

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Concurrence: $\mathcal{C}^{\text{out}} = 0.894$

Real part

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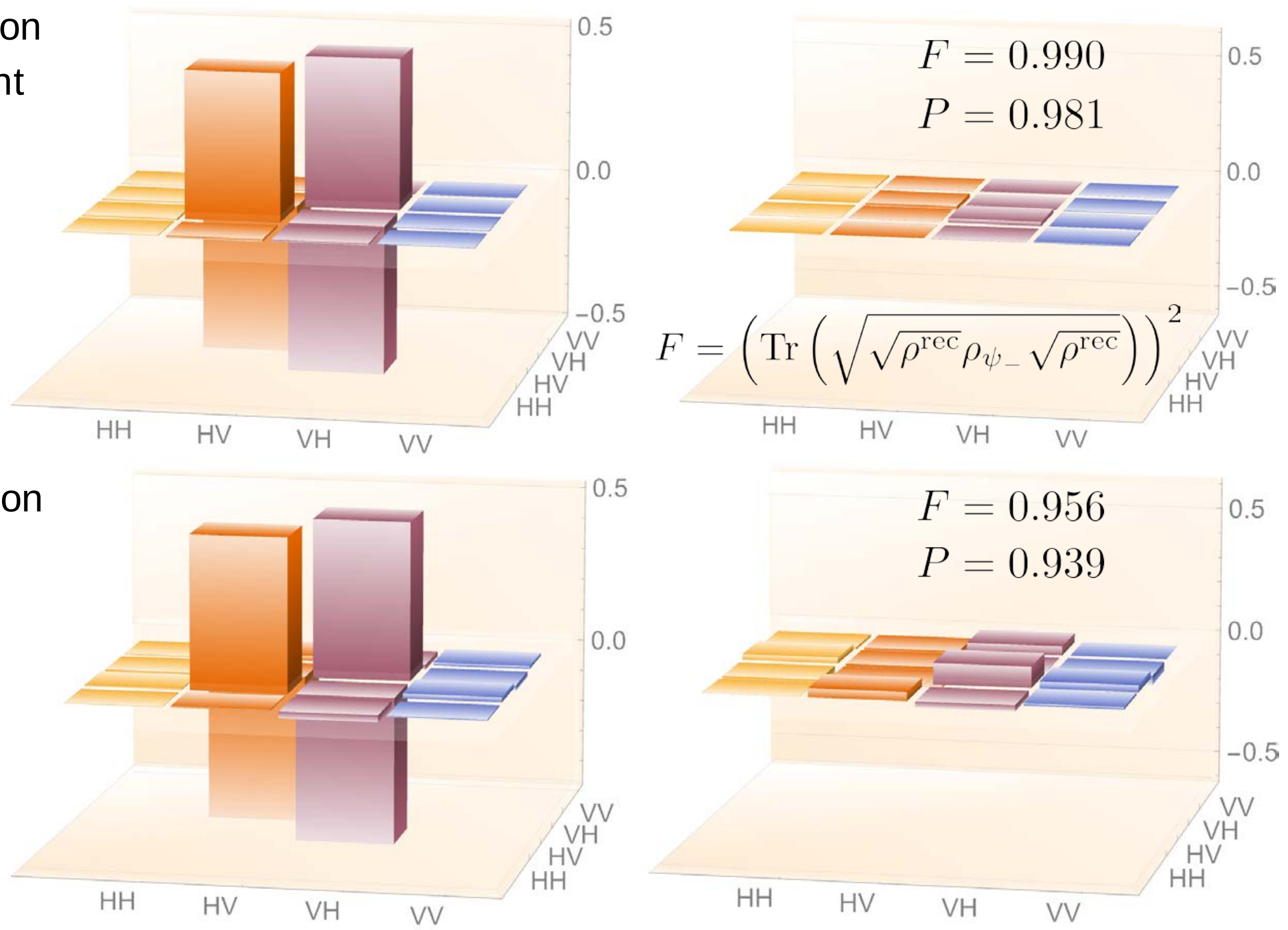

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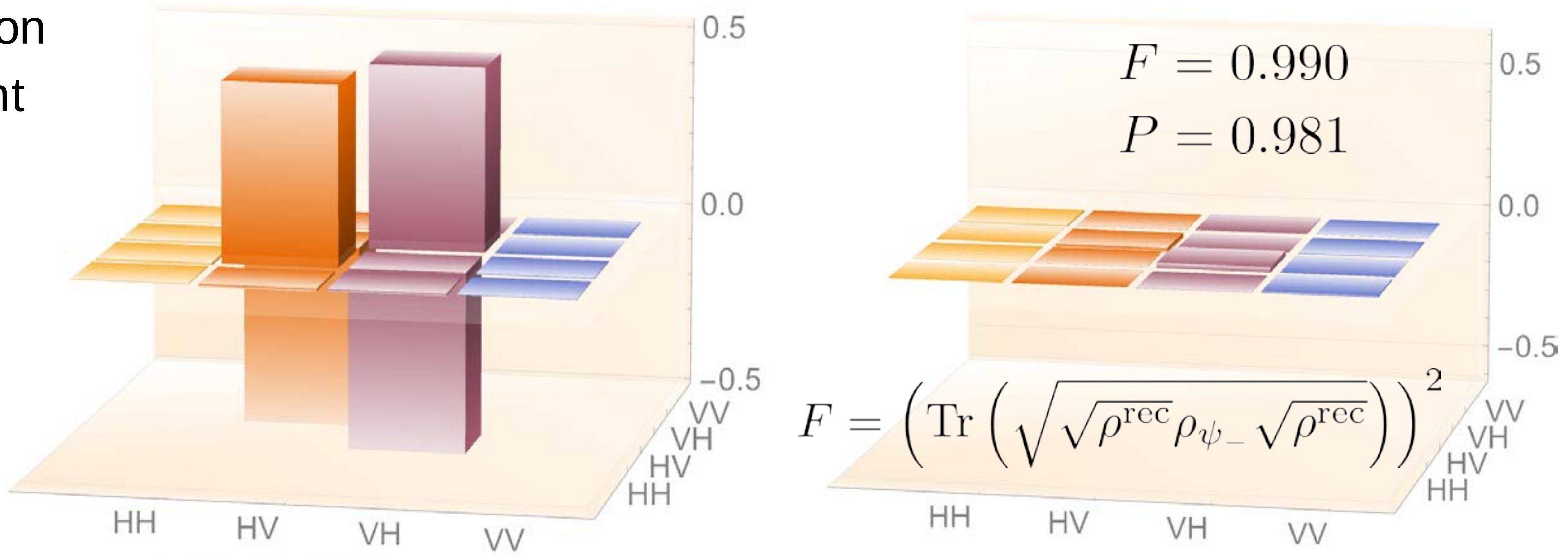

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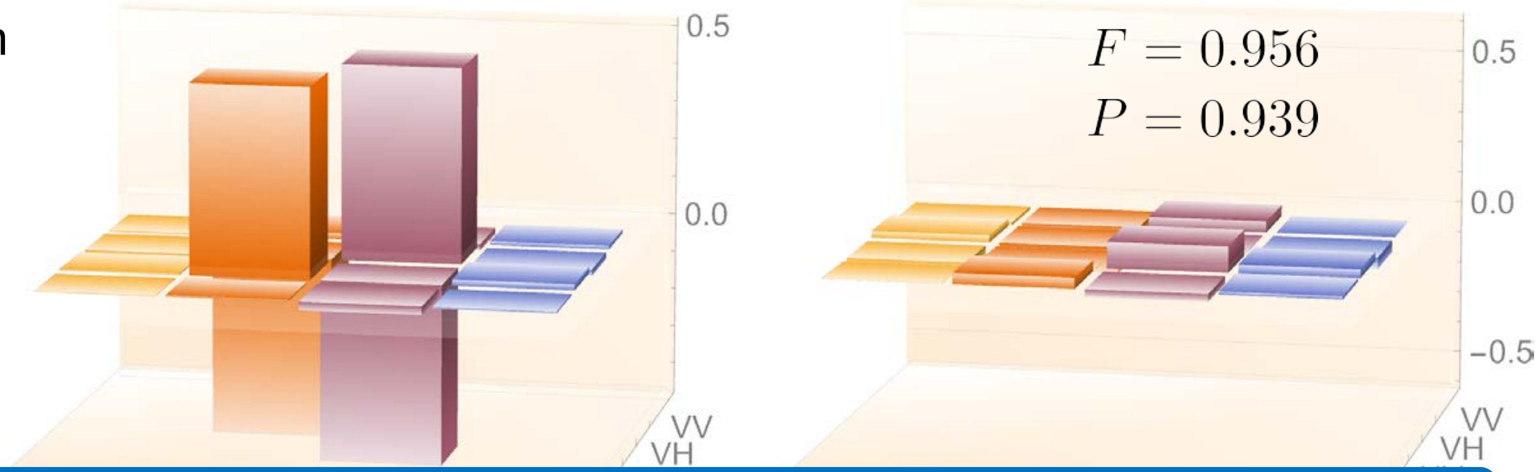

Our measurement procedure induces just a tiny decoherence on the initial state: **the entanglement is still here!**

Summary

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- Next step: implementation of novel quantum foundations investigation tests involving measurements on non-commuting observables, e.g. the one connected to the Relativistic Independence condition [Carmi & Cohen, Sci. Adv. 5, eaav8370 (2019)].

Credits

Salvatore Virzì
Enrico Rebufello
Francesco Atzori
Fabrizio Piacentini
Alessio Avella
Marco Gramegna
Ivo P. Degiovanni
Marco Genovese

INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA

Eliahu Cohen

Bar-Ilan University

Rudi Lussana
Iris Cusini
Francesca Madonini
Federica Villa

POLITECNICO MILANO 1863

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Quantum Physics

[Submitted on 8 Mar 2023]

Single-pair measurement of the Bell parameter

Salvatore Virzì, Enrico Rebufello, Francesco Atzori, Alessio Avella, Fabrizio Piacentini, Rudi Lussana, Iris Cusini, Francesca Madonini, Federica Villa, Marco Gramegna, Eliahu Cohen, Ivo Pietro Degiovanni, Marco Genovese

Bell inequalities are one of the cornerstones of quantum foundations, and fundamental tools for quantum technologies. Recently, the scientific community worldwide has put a lot of effort towards them, which culminated with loophole-free experiments. Nonetheless, none of the experimental tests so far was able to extract information on the full inequality from each entangled pair, since the wave function collapse forbids performing, on the same quantum state, all the measurements needed for evaluating the entire Bell parameter. We present here the first single-pair Bell inequality test, able to obtain a Bell parameter value for every entangled pair detected. This is made possible by exploiting sequential weak measurements, allowing to measure non-commuting observables in sequence on the same state, on each entangled particle. Such an approach not only grants unprecedented measurement capability, but also removes the need to choose between different measurement bases, intrinsically eliminating the freedom-of-choice loophole and stretching the concept of counterfactual-definiteness (since it allows measuring in the otherwise not-chosen bases). We also demonstrate how, after the Bell parameter measurement, the pair under test still presents a noteworthy amount of entanglement, providing evidence of the absence of (complete) wave function collapse and allowing to exploit this quantum resource for further protocols.

Comments: 14 pages, 3 figures
Subjects: **Quantum Physics (quant-ph)**
Cite as: arXiv:2303.04787 **[quant-ph]**
(or arXiv:2303.04787v1 **[quant-ph]** for this version)
https://doi.org/10.48550/arXiv.2303.04787

Weak values

Weak measurements [Aharonov, Albert & Vaidman, PRL 60 (1988)]: little information is extracted from a single measurement event, but the state does NOT collapse.

Weak value: $\langle \widehat{A} \rangle_w = \dfrac{\langle \varphi | \widehat{A} | \psi \rangle}{\langle \varphi | \psi \rangle}$

$|\psi\rangle$: pre-selected state

$|\varphi\rangle$: post-selected state

Von Neumann coupling between an observable $\hat{A}$ and a pointer observable $\hat{P}$: $\widehat{U} = \exp(-ig\widehat{A} \otimes \widehat{P})$

Projective measurement (post-selection on $|\varphi\rangle$): $\widehat{\Pi}_\varphi = |\varphi\rangle\langle\varphi|$ $\Rightarrow$ $|\Psi_{out}\rangle = \widehat{\Pi}_\varphi \widehat{U} |\Psi\rangle = \widehat{\Pi}_\varphi \widehat{U} |\psi\rangle \otimes |\phi_p\rangle$

Weak interaction (1° order approximation): $\Rightarrow$ $\langle \widehat{X} \rangle = \dfrac{\langle \Psi_{out} | \widehat{X} | \Psi_{out} \rangle}{\langle \psi | \widehat{\Pi}_\varphi | \psi \rangle} = g \ \mathrm{Re}[\langle \widehat{A} \rangle_w]$

$\widehat{X}$ and $\widehat{P}$ canonically conjugated

Joint and Sequential Weak Values

Weak values *«challenge one of the canonical dicta of QM: that non commuting observables cannot be simultaneously measured»*

«the fact that one hardly disturbs the systems in making WM means that one can in principle measure different variables in succession» [Mitchison, Jozsa and Popescu, PRA 76 (2007)]

Joint weak measurement

Resch et al., PRL 92, 130402 (2004)

$$\widehat{U} = \exp[-i(g_x \widehat{A} \otimes \widehat{P}_x + g_y \widehat{B} \otimes \widehat{P}_y)]$$

$$\langle \widehat{X}\widehat{Y} \rangle = \frac{1}{4} g_x g_y \mathrm{Re}\left[\langle \widehat{A}\widehat{B} + \widehat{B}\widehat{A} \rangle_w + 2\langle \widehat{A} \rangle_w^* \langle \widehat{B} \rangle_w \right]$$

Sequential weak measurement

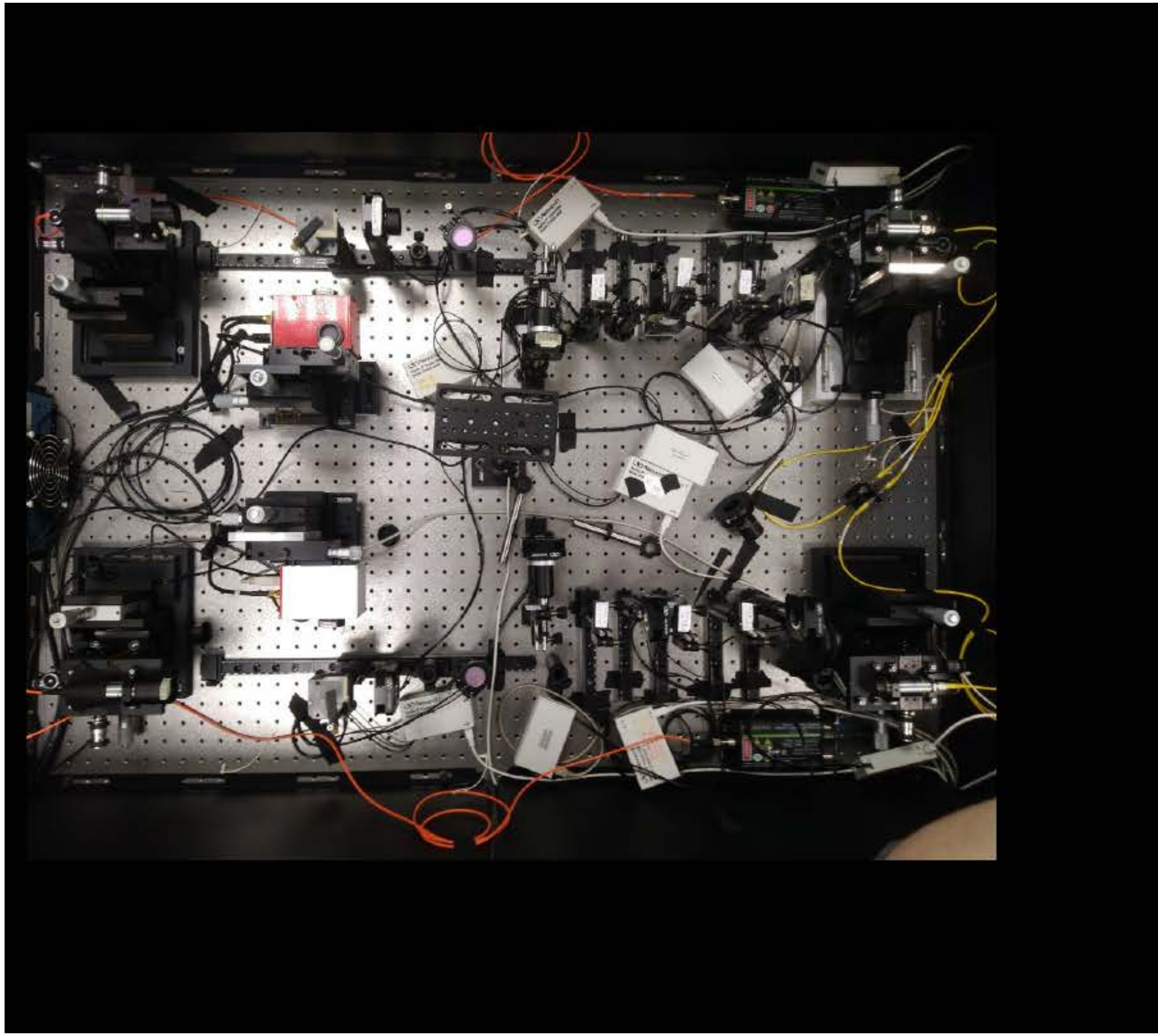
Mitchinson et al., PRA 76, 062105 (2007)

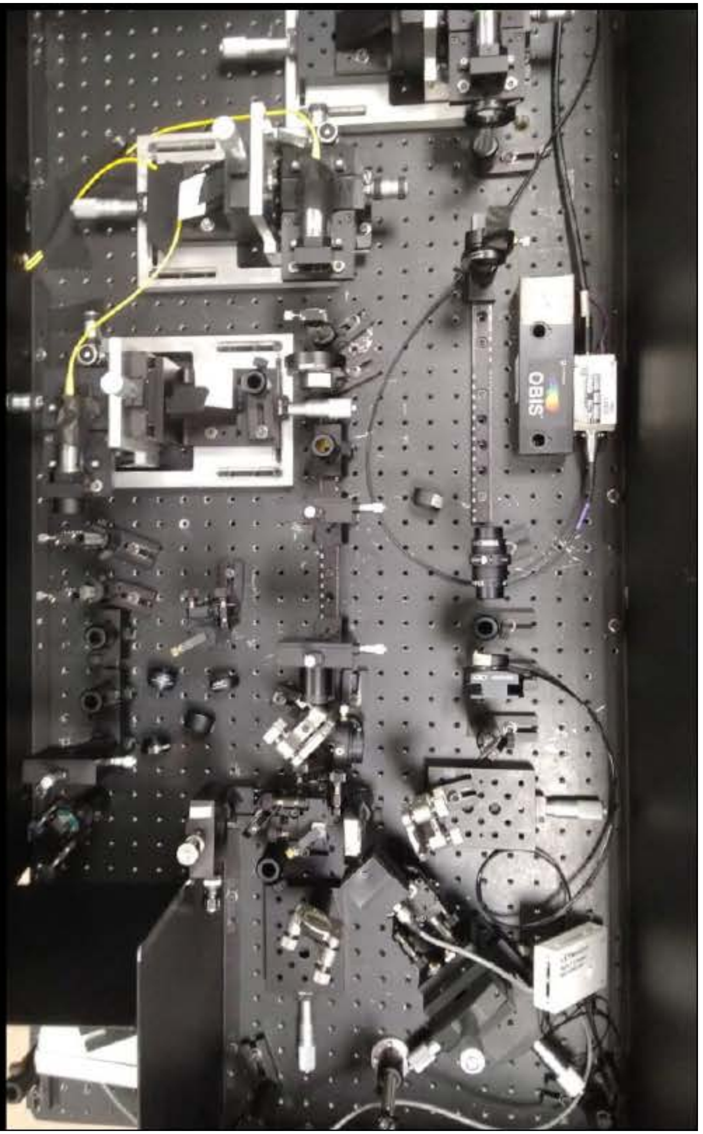
$$\widehat{U}_y = \exp(-i g_y \widehat{B} \otimes \widehat{P}_y)$$

$$\widehat{U}_x = \exp(-i g_x \widehat{A} \otimes \widehat{P}_x)$$

$$\langle \widehat{X}\widehat{Y} \rangle = \tfrac{1}{2} g_x g_y \mathrm{Re}\left[\langle \widehat{A}\widehat{B} \rangle_w + \langle \widehat{A} \rangle_w^* \langle \widehat{B} \rangle_w \right]$$

Detailed experimental setup





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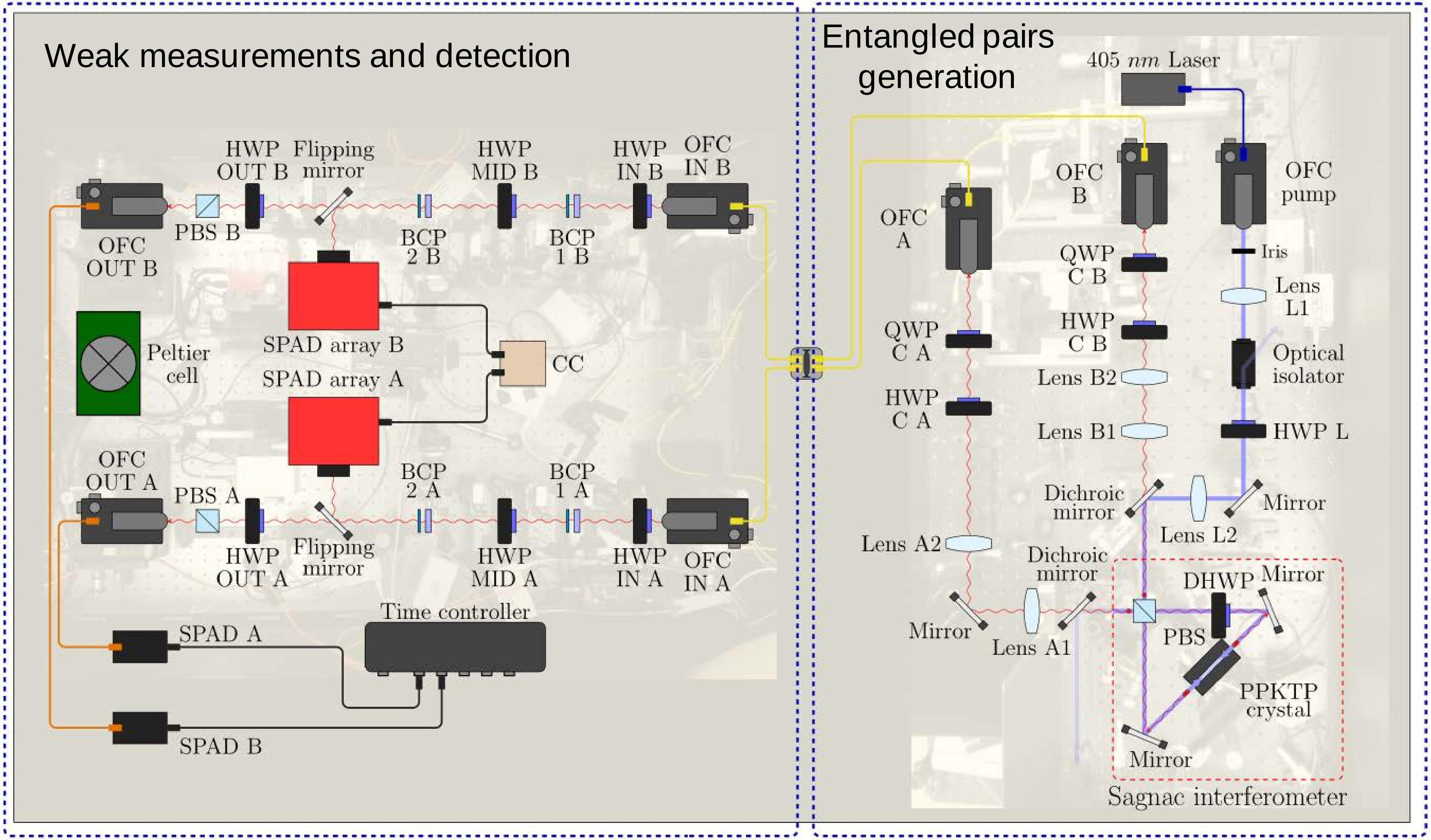

Negativity and Concurrence estimation

Negativity:

$$\mathcal{N}(\rho) = ||\rho^{\Gamma_A}||_1 - 1$$

ρ_A^{Γ}: partial transpose of ρ with respect to the subsystem A

$||X||_1 = \mathrm{Tr}\left(\sqrt{X^\dagger X}\right)$: trace norm of the operator X

Concurrence:

$$\mathcal{C}(\rho) = \mathrm{Max}(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

λ_i : R eigenvalues

$$R = \sqrt{\sqrt{\rho}(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\sqrt{\rho}}$$

We suppose for our state a density matrix ρ:

$$\rho = p\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2\theta & -\cos\theta\sin\theta & 0 \\ 0 & -\cos\theta\sin\theta & \sin^2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} (1-p)\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2\theta & 0 & 0 \\ 0 & 0 & \sin^2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{N}^{\mathrm{in}} = \mathcal{C}^{\mathrm{in}} = 0.983 \pm 0.001$$

$$\mathcal{N}^{\mathrm{out}} = \mathcal{C}^{\mathrm{out}} = 0.927 \pm 0.001$$

Negativity and Concurrence can be optimally estimated as [Virzì et al., Sci. Rep. 9, 3030 (2019)]:

$$\mathcal{N}(\rho) = \mathcal{C}(\rho) = (P(|+-\rangle) + P(|-+\rangle) - P(|++\rangle) - P(|--\rangle))$$