

# COOPERATIVE QUANTUM INFORMATION ERASURE

— Michele Campisi —

CNR - Nano Science Institute  
& Scuola Normale Superiore



quantum annealing time  
provided by



project q-lend



Lorenzo  
Buffoni (Florence)

## Cooperative quantum information erasure

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Doi: <https://doi.org/10.22331/q-2023-03-23-qf1>

Citation: Quantum 7, 961 (2023).



Qubit initialisation  
(quantum information  
erasure)

any  $\rho$   $\rightarrow$   $1 \times 1$   
pure

Energetic cost :

Piechocinska, PRE 61:062314 (2000)

Esposito et al, NJP 12:013013 (2010)

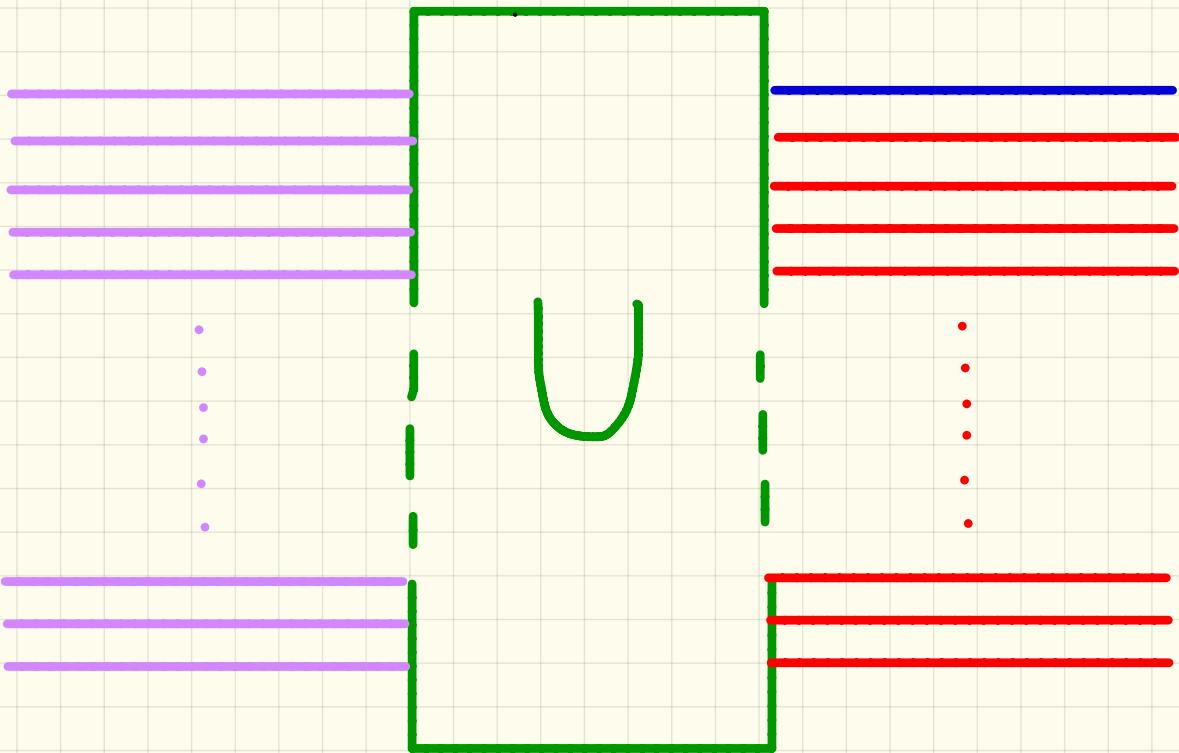
$$W \geq T \Delta S$$

$$\Delta S = -T \text{Tr } \rho \ln \rho$$

$$\left( \begin{array}{ll} \rho = \text{Id}/2 & \Delta S = \ln 2 \end{array} \right)$$

$T$  = environment  
Temperature

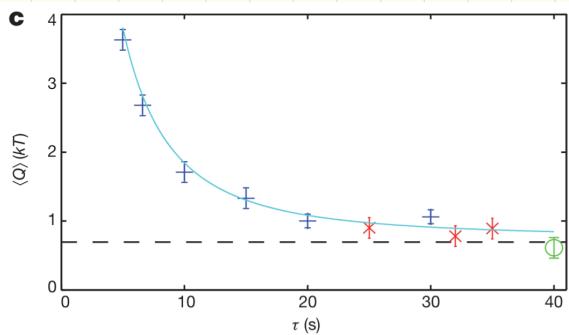
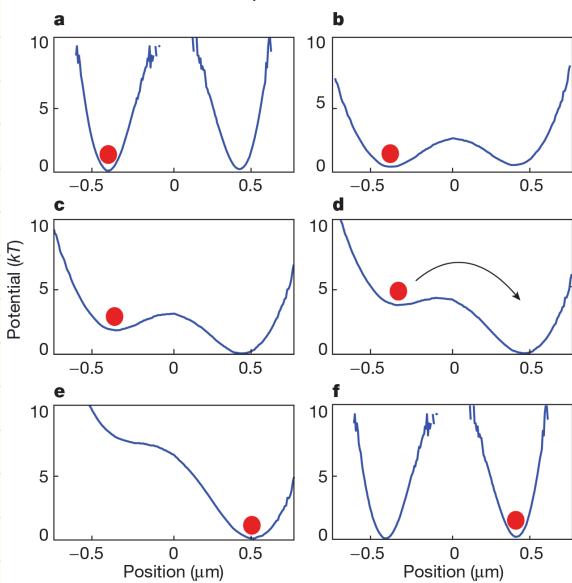
standard / quantum info) approach: algorithmic cooling  
(quantum thermo) (quantum heat engine)



Landauer (1961)

minimal energy cost of writing (=resetting)  
1 bit of information

$$W = kT \ln 2$$



Berut et al. Nature 483 (7388)

187-189 (2012)

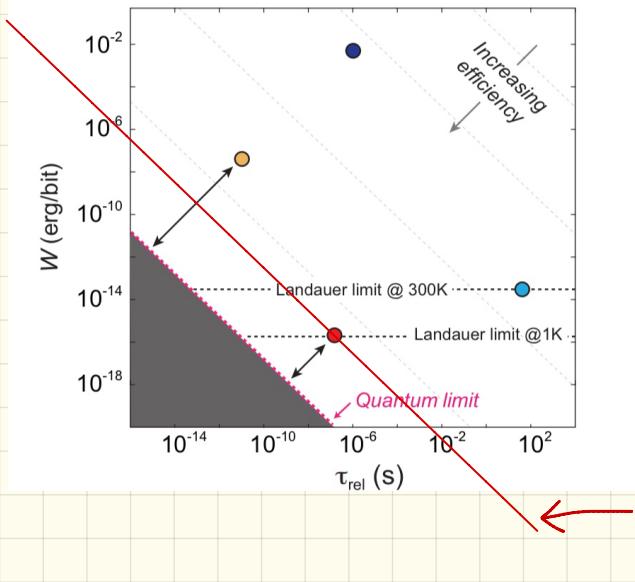
figure of merit = erasing action

$$A = w \cdot T$$

work per bit      erasure time

Gaudenzi et al., Nat. Phys. 14 565 (2018)

- Fe<sub>8</sub> Single molecule magnet
- Bead in optical trap
- GdFeCo 20 nm<sup>3</sup> cells
- Flip-Flop device

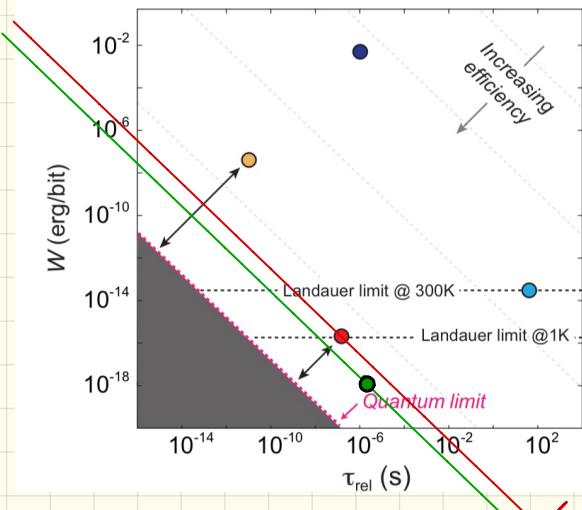


$\sim 10^{-22} \frac{\text{erg} \cdot \text{s}}{\text{bit}}$

Gaudenzi et al,  
Nat. Phys. 14  
565 (2018)

- Fe<sub>8</sub> Single molecule magnet
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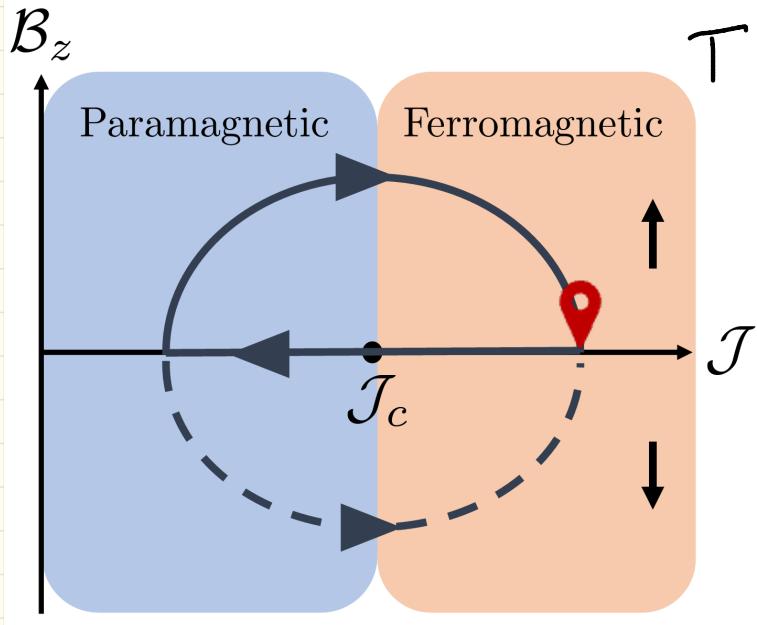
● this work



$\sim 10^{-23} \frac{\text{erg}\cdot\text{s}}{\text{bit}}$  → this work ●

$\sim 10^{-22} \frac{\text{erg}\cdot\text{s}}{\text{bit}}$  → Gaudenzi et al.,  
Nat. Phys. 14  
565 (2018)

# OUR MAIN IDEA:



$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - B_z \sum_i \sigma_i^z$$

SPONTANEOUS SYMMETRY BREAKING + QUANTUM TUNNELING

## Implementation : D-Wave 4.1 Advantage processor

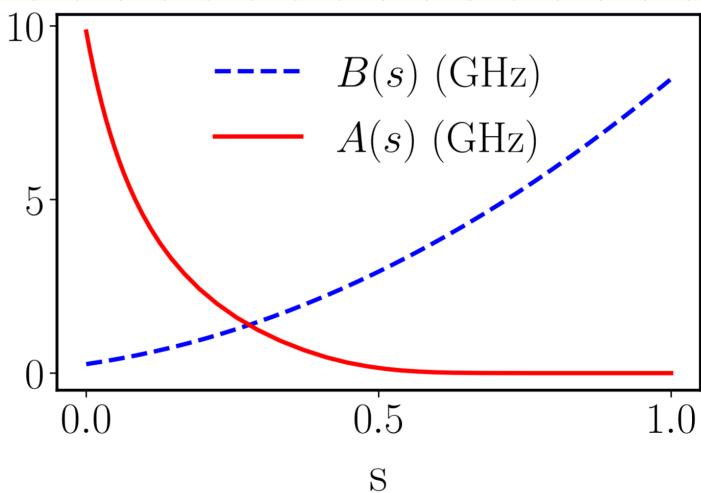
$$\frac{H(s)}{h} = -\frac{\underline{A}(s)}{2} \sum_i \sigma_i^x - \frac{\underline{B}(s)}{2} \left[ g \sum_i \sigma_i^z + \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z \right]$$

$\zeta \rightarrow \zeta(t)$

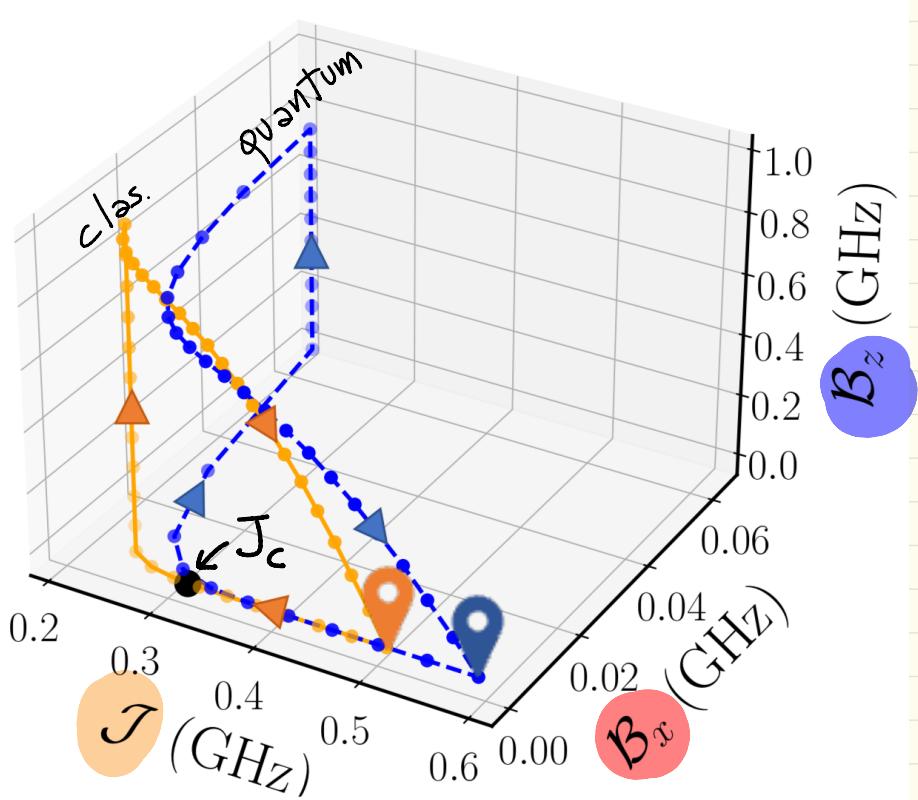
$g \rightarrow g(t)$

$J_{ij} \rightarrow \text{constant}$

$$N = 16 \times 16 = 256$$



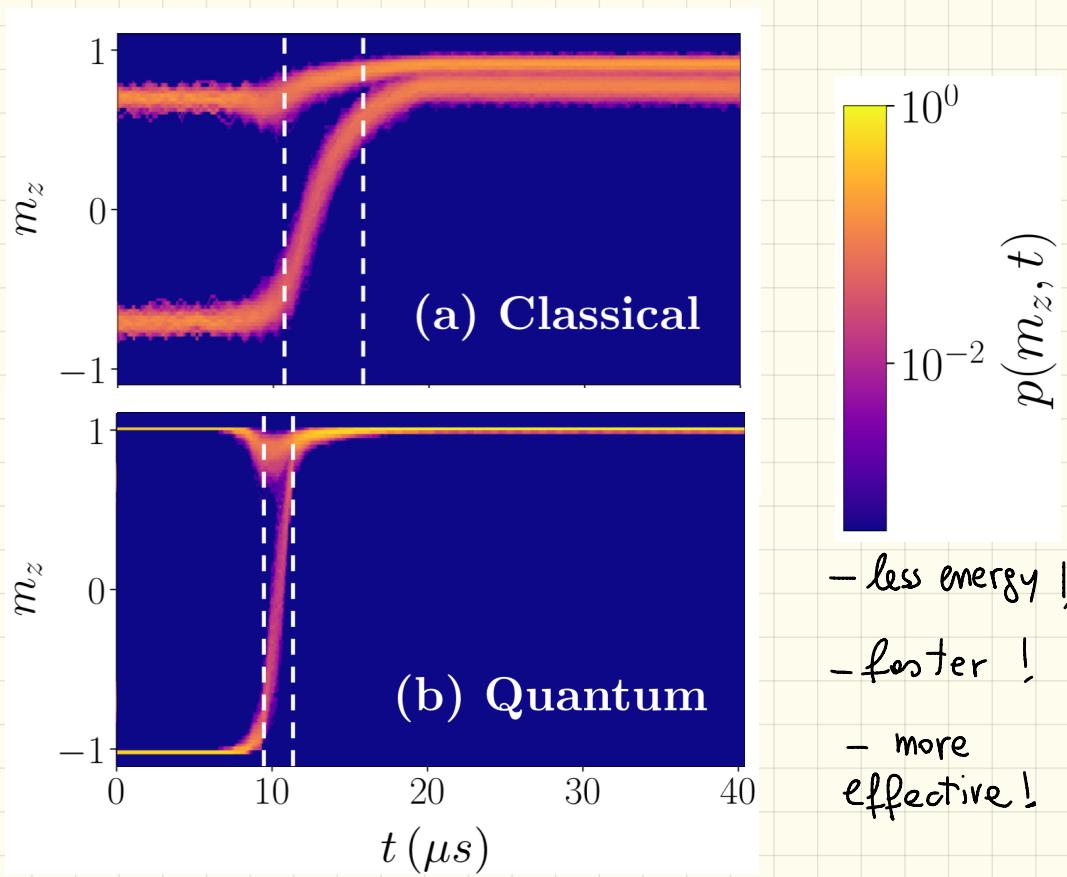
$$\frac{H(t)}{h} = -\mathcal{B}_x(t) \sum_i \sigma_i^x - \mathcal{B}_z(t) \sum_i \sigma_i^z - \mathcal{J}(t) \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z$$



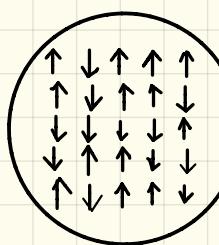
# erasure of a single macroscopic bit

$$\mathbb{M}_z = \frac{\sum_i s_i^z}{N}$$

$$\left( \begin{array}{l} \langle \sigma_i^z | s_i^z \rangle = s_i^z | s_i^z \rangle \\ s_i^z = \pm 1 \end{array} \right)$$

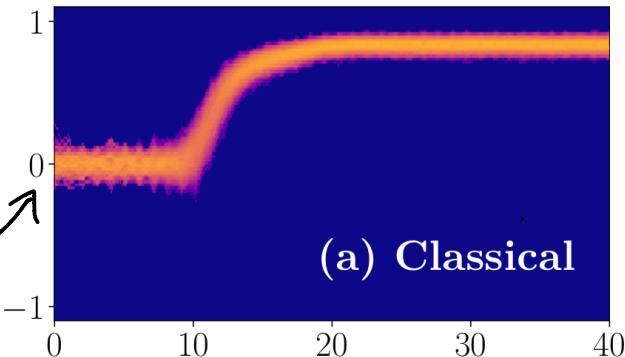


# coop quantum erasure of $N$ bits



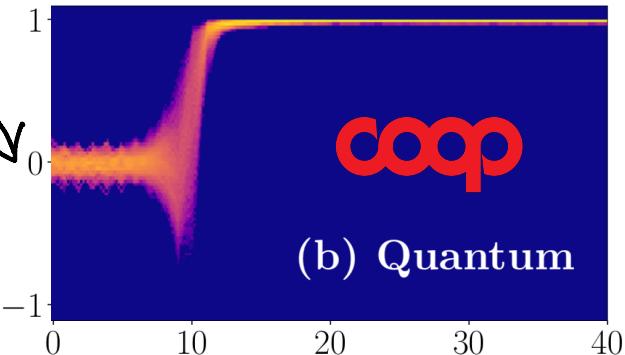
50 %

$m_z$

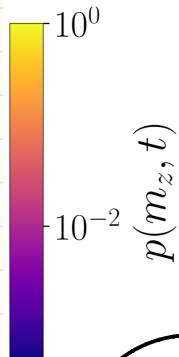


(a) Classical

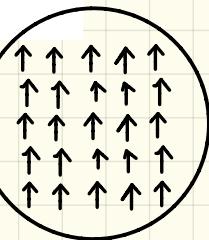
$m_z$



(b) Quantum



$p(m_z, t)$



99,9 %  
Success

$T_{\text{spin}} = \infty$

cooling

$\Rightarrow T_{\text{spin}} \approx 0$

## Energy Cost

$$\begin{aligned} W &= \langle H^{\text{TOT}}(\tau) \rangle - \langle H^{\text{TOT}}(0) \rangle = \int_0^\tau dt \frac{d}{dt} \langle H^{\text{TOT}} \rangle = \int_0^\tau dt \langle \frac{\partial H^{\text{TOT}}}{\partial t} \rangle \\ &= \int_0^\tau dt \langle \frac{\partial H}{\partial t} \rangle = \oint d\underline{R} \langle \frac{\partial H}{\partial \underline{R}} \rangle \\ \underline{R} &= (\underbrace{B_x}_{\text{red}}, \underbrace{B_z}_{\text{blue}}, \underbrace{J}_{\text{orange}}) \quad \frac{\partial H}{\partial \underline{R}} = (\underbrace{M_x}_{\text{red}}, \underbrace{M_y}_{\text{blue}}, \underbrace{K}_{\text{orange}}) \end{aligned}$$

$$M_x = \sum_i \langle \sigma_i^x \rangle, M_z = \sum_i \langle \sigma_i^z \rangle, K = \sum_{\langle i,j \rangle} \langle \sigma_i^z \sigma_j^z \rangle$$

$$W = - \int_C (\underbrace{M_x d\mathcal{B}_x}_{\text{red}} + \underbrace{M_z d\mathcal{B}_z}_{\text{blue}} + \underbrace{K d\mathcal{J}}_{\text{orange}})$$

$$\doteq \underbrace{W_x}_{\text{red}} + \underbrace{W_z}_{\text{blue}} + \underbrace{W_{zz}}_{\text{orange}}.$$

$W_z, W_{zz} \rightarrow$  directly from the  $s_i^z$

$W_x \rightarrow$  cannot access  $s_i^x$ : bound its modulus

$$|M_x| = \left| \sum_i \langle \sigma_i^x \rangle \right| \leq \sum_i |\langle \sigma_i^x \rangle| \leq \sum_i \sqrt{1 - \langle \sigma_i^z \rangle^2} \doteq M_*$$

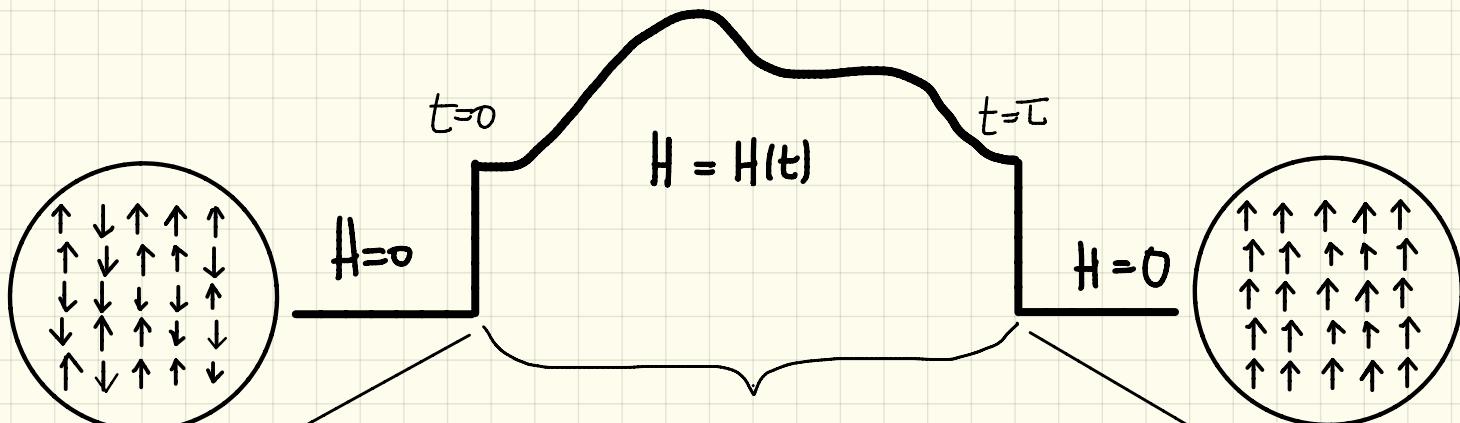
$F = \text{forward}$   
 $B = \text{backward}$

$$\begin{aligned} W_x &= - \int_{\mathcal{B}_x^{\min}}^{\mathcal{B}_x^{\max}} M_x^F d\mathcal{B}_x - \int_{\mathcal{B}_x^{\max}}^{\mathcal{B}_x^{\min}} M_x^B d\mathcal{B}_x \\ &= - \int_{\mathcal{B}_x^{\min}}^{\mathcal{B}_x^{\max}} (M_x^F - M_x^B) d\mathcal{B}_x \leq \int_{\mathcal{B}_x^{\min}}^{\mathcal{B}_x^{\max}} (|M_x^F| + |M_x^B|) d\mathcal{B}_x \\ &= \int_{\mathcal{B}_x^{\min}}^{\mathcal{B}_x^{\max}} (M_*^F + M_*^B) d\mathcal{B}_x \doteq \delta W. \end{aligned}$$

Same for  $-W_x$   
 $\Rightarrow |W_x| \leq \delta W$

$$W_{\text{exp}} = W_z + W_{zz} \pm \delta W \rightarrow \text{from } s_i^z$$

$$\frac{H(t)}{h} = -\mathcal{B}_x(t) \sum_i \sigma_i^x - \mathcal{B}_z(t) \sum_i \sigma_i^z - \mathcal{J}(t) \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z$$

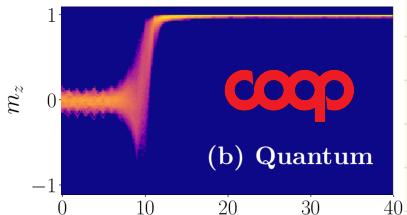


$$W=0$$

$$W = W_z + W_{zz} + W_x$$

$$W = -\nabla \phi$$

$$w = 3,55(0,73) \times 10^{-18} \text{erg/bit}$$



$$w_L = kT \ln 2 = 3,7(0,8) \times 10^{-18} \text{erg/bit}$$

( $T = 39 \pm 9 \text{ mK}$ )

$$T_{99,9} \approx 9 \text{ } \mu\text{s}$$

$$\mathcal{A}_{99,9\%} = 3.19(0.27) \times 10^{-23} \text{erg} \cdot \text{s}/\text{bit}$$

## Remarks

- 1) Erasure occurs @ minimal energy cost (within error)
- 2) Lowest erasure action reported to date  
To the best of our knowledge
- 3) Very high success rate : 99,9 %
- 4) Very stable bit reset : at least order of seconds
- 5) Not optimised → room for improvement

## Remarks

- 6) Erase all  $N$  qubits at once ( algorithmic cooling erases a fraction )
- 7) Expected to work the better the larger  $N$  !!
- 8) Application: initialisation of quantum processing units
- 9) Openness was crucial for achieving purification  
"Mixedness" was dump into environment!

.. all this thanks to a shift in cooling paradigm

# SYNERGY OF COOPERATIVE EMERGENT MANY-BODY EFFECTS (spontaneous Symmetry breaking)

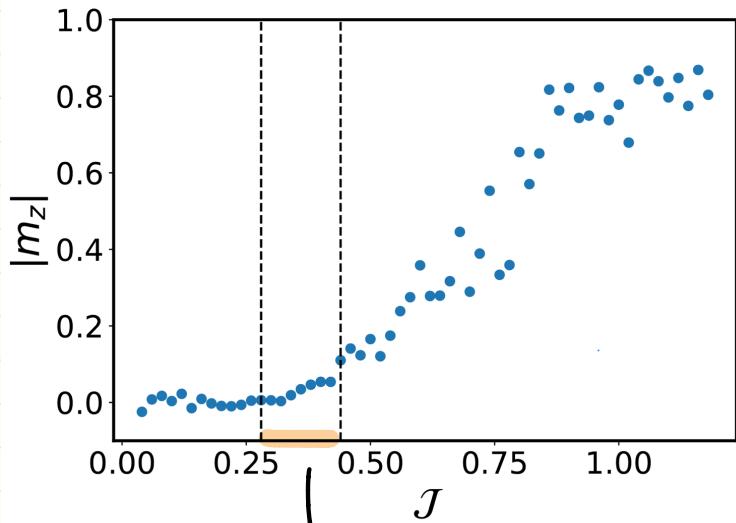


## QUANTUM EFFECTS

Thanks



# Estimating the Temperature



$J_c$

$$S(t) = \frac{t}{\tau} \quad T = 200 \mu s$$

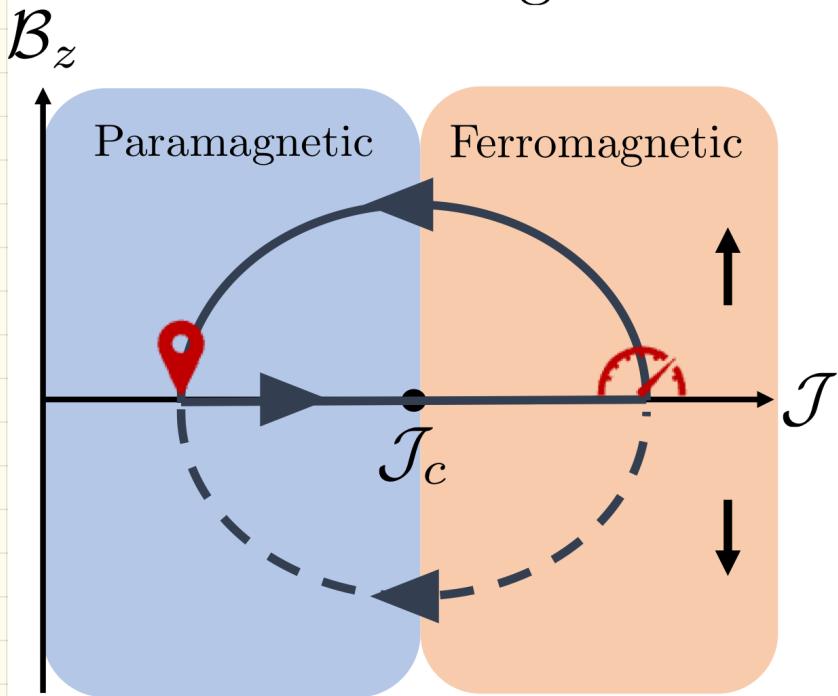
@  $f=0$

$$\frac{H(s)}{h} = -\frac{A(s)}{2} \sum_i \sigma_i^x - \frac{B(s)}{2} \left[ \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z \right]$$

$$kT = \frac{2 J_c}{\ln(1 + \sqrt{2})}$$

$\text{erg} \times 10^{-18}$	Classical er.	Quantum er.	Qu. coop. er.
$W_z$	1067(80)	331(66)	166(60)
$W_{zz}$	-351(86)	-9.3(13)	-1140(20)
$\delta W$	36	53	106
$U_f$			-1884
$W_{\text{exp}}$	716(202)	322(132)	910(186)
$W_L$	3.71(0.79)	3.71(0.79)	949(202)

# Szilard Engine



(a)

J. Parrondo,  
Chaos 11, 725 (2001)

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - B_z \sum_i \sigma_i^z$$

# Quantum energy advantage ?

Quantum Technologies Need a Quantum Energy Initiative

Alexia Auffèves

PRX Quantum **3**, 020101 – Published 1 June 2022

PRX QUANTUM  
*a Physical Review journal*

<https://quantum-energy-initiative.org>