Quantum Sailing Conference 14-20 May 2023

Fluctuation Relation and Irreversibility Mitigation in Thermalizing Quantum Dynamics

Eliana Fiorelli

IFISC Universitat de les Illes Balears CSIC

Stefano Gherardini

CNR-INO Istituto Nazionale di Ottica Consiglio Nazionale delle Ricerche

Stefano Marcantoni

SISSA

OUTLINE

1. STOCHASTIC ENTROPY PRODUCTION

   two point measurement scheme

2. FLUCTUATION RELATION

   under non-unital dynamics

3. IRREVERSIBILITY MITIGATION

   qubit thermalizing dynamics

TWO POINT MEASUREMENT SCHEME

S , Hilbert space $\mathcal{H}_S$, $\dim \mathcal{H}_S < +\infty$

- evolution $\{\Lambda_t\}$ CPTP map $\Lambda_t(\cdot) = \sum_\ell E_\ell(t)\,(\cdot)\,E_\ell^\dagger(t)$

- measurements $\begin{cases} O_{in} = \sum_m a_m^{in}\, \Pi_m^{in} \\ O_{fin} = \sum_k a_k^{fin}\, \Pi_k^{fin} \end{cases}$

TPM : measurements – evolution – measurements

FORWARD PROTOCOL

$$\rho_0 \xrightarrow{\{\Pi_m^{in}\}} \rho_{in} \xrightarrow{\Lambda_t} \rho_t \xrightarrow{\{\Pi_k^{fin}\}} \rho_{fin}$$

joint probability $$p_F(a_k^{fin}, a_m^{in}) = \mathrm{Tr}\left[\Pi_k^{fin} \Lambda_t(\Pi_m^{in})\right] p(a_m^{in})$$

Reversed path ?

BACKWARD PROTOCOL

- Θ time reversal
- $\pi : \Lambda_t(\pi) = \pi$ unique invariant state
- $\tilde{\Lambda}_t(\cdot) = \sum_\ell \tilde{E}_\ell(t)\,(\cdot)\,\tilde{E}_\ell^\dagger(t)$ dual map, $\tilde{E}_\ell(t) = \Theta\, \pi^{1/2} E_\ell^\dagger(t)\, \pi^{-1/2}\, \Theta^\dagger$

$$\tilde{\rho}_{fin} \xrightarrow{\{\tilde{\Pi}_k^{fin}\}} \tilde{\rho}_t \xrightarrow{\tilde{\Lambda}_t} \tilde{\rho}_{in} \xrightarrow{\{\tilde{\Pi}_m^{in}\}} \tilde{\rho}_0$$

joint probability $\quad p_B(a_m^{in}, a_k^{fin}) = \mathrm{Tr}\left[\tilde{\Pi}_m^{in}\, \tilde{\Lambda}_t(\tilde{\Pi}_k^{fin})\right] p(a_k^{fin})$

[Crooks PRA 77 034101 2008]

Stochastic entropy production

- $$\Delta\sigma(k,m) = \ln\left(\frac{P_F}{P_B}\right) = \ln\left[\frac{\phi(a_m^{in})}{\phi(a_k^{fin})}\right] + \underbrace{\ln\left[\frac{\phi_F(a_k^{fin}|a_m^{in})}{\phi_B(a_m^{in}|a_k^{fin})}\right]}$$

  0 for unital maps

  in general depends on details of Λ_t

- $$\text{Prob}(\Delta\sigma) = \sum_{k,m} \delta[\Delta\sigma - \Delta\sigma(k,m)]\, \phi_F(a_k^{fin}, a_m^{in})$$

NONEQUILIBRIUM POTENTIAL

- $\Lambda_t(\pi) = \pi \qquad \pi = \sum_i \pi_i |\pi_i\rangle\langle\pi_i|$

  to each $|\pi_i\rangle\langle\pi_i|$ assign $\Phi_\pi(i) = -\ln(\pi_i)$

- Assumptions:

  (a) Kraus operators $E_\ell(t) = \sum_{ij} m^\ell_{ji}(t) |\pi_j\rangle\langle\pi_i|$

  with the constraint $m^\ell_{ji}(t) = 0 \quad \forall t$ if $\Phi_\pi(j) - \Phi_\pi(i) \neq \Delta\Phi_\pi(\ell)$

  (b) Measurement (O_{in}, O_{fin}) $\quad \Pi^{in}_m = |\pi_m\rangle\langle\pi_m|$

  $\Pi^{fin}_k = |\pi_k\rangle\langle\pi_k|$

[G. Manzano et al PRE 92 032129 (2015)]

FLUCTUATION RELATION

Under assumptions (a), (b):

- S.E.P $$\Delta\sigma(k,m) = \ln\left[\frac{\phi(a_m^{in})}{\phi(a_k^{fin})}\right] + \ln\left[\frac{\phi_F(a_k^{fin}|a_m^{in})}{\phi_B(a_m^{in}|a_k^{fin})}\right] = \Delta S(k,m) - \Delta\Phi_\pi(k,m)$$

- F.R. $$\frac{\phi_F(a_k^{fin}, a_m^{in})}{\phi_B(a_m^{in}, a_k^{fin})} = e^{\Delta\sigma(k,m)} = e^{\Delta S(k,m) - \Delta\Phi_\pi(k,m)}$$

STATISTIC OF SEP

Average $\langle \Delta\sigma \rangle = S(\rho_t \| \rho_{fin}) + S(\rho_{in} \| \pi) - S(\rho_t \| \pi) \geq 0$

$S(\rho \| \rho') = \mathrm{Tr}[\rho \ln \rho - \rho \ln \rho']$ quantum relative entropy

Variance $\mathrm{Var}(\Delta\sigma)$

IRREVERSIBILITY MITIGATION : If the dynamics displays "strong" non-markovianity it is possible to find time-intervals both $\langle \Delta\sigma \rangle$, $\mathrm{Var}(\Delta\sigma)$ decreasing

NON MARKOVIAN DYNAMICS

$\Lambda_t = V_{t,s} \circ \Lambda_s$, $t \geq s \geq 0$, dynamics is :

- CP-divisible (markovian) if $V_{t,s}$ CP
- P-divisible (weakly markovian) if $V_{t,s}$ $\cancel{\text{C}}$P
- not P-divisible (essentially non-markovian) if $V_{t,s}$ $\cancel{\text{CP}}$

If Λ_t invertible $V_{t,s} = \Lambda_t \circ \Lambda_s^{-1}$

$$\mathcal{L}_t = (\partial_t \Lambda_t)\, \Lambda_t^{-1}$$ time-dependent generator

[D. Chruściński, S. Maniscalco, PRL 112 120404 (2014)]

QUBIT THERMALIZATION

$$\mathcal{L}_t(\bullet) = -i\left[\frac{\omega}{2}\hat{\sigma}_z, \bullet\right] + \gamma_\beta(t)e^{\omega\beta}\left(\hat{\sigma}_- \bullet \hat{\sigma}_+ - \frac{1}{2}\{\hat{\sigma}_+\hat{\sigma}_-, \bullet\}\right) + \gamma_\beta(t)\left(\hat{\sigma}_+ \bullet \hat{\sigma}_- - \frac{1}{2}\{\hat{\sigma}_-\hat{\sigma}_+, \bullet\}\right)$$

$\Lambda_t(\cdot)$ is CP iff $\Gamma_\beta(t) = (1+e^{\beta\omega})\int_0^t \gamma_\beta(\tau)d\tau \geq 0$

$\Lambda_t(\cdot)$ is P & CP-divisible iff $\gamma_\beta(t) \geq 0 \quad \forall t > 0$

$\Lambda_t(\cdot)$ Kraus representation satisfies condition (a)

$$E_{1,2} = \alpha_{1,2}\hat{\sigma}_{+,-} \qquad E_{3,4} = \eta_{3,4}|0\rangle\langle 0| + \bar{\eta}_{3,4}|1\rangle\langle 1|$$

IRREVERSIBILITY MITIGATION

- Measure $\hat{\sigma}_z$ [condition (b)], and prepare $|0\rangle\langle 0|$

$$\partial_t \langle \Delta\sigma \rangle = -\frac{1}{2} I(t)\, \partial_t z(t) \quad \begin{cases} \partial_t z(t) = -2\, \partial_t \Gamma_\beta(t)\, e^{-2\Gamma_\beta(t)} (1 - z_\infty) \\ I(t) = \ln\left[\frac{1 + z(t)}{1 - z(t)}\right] + \beta\omega \geq 0 \end{cases}$$

$\partial_t \langle \Delta\sigma \rangle < 0$ iff $\gamma_\beta(t) < 0$

$$\partial_t \mathrm{Var}(\Delta\sigma) = 2\, \partial_t \langle \Delta\sigma \rangle \left(\frac{z(t)}{2} I(t) - 1 \right) \quad \begin{cases} \ldots \text{ some algebra} \\ \Gamma_\beta(t) \leq -\frac{1}{2} x_+(\beta), \quad x_+(\beta) = \ln\left[\frac{2}{5}\left(1 - e^{-\omega\beta} + \sqrt{e^{-2\beta\omega} + 3e^{-\beta\omega} + 1}\right)\right] \end{cases}$$

- $\partial_t \langle \Delta\sigma \rangle < 0$, $\partial_t \mathrm{Var}(\Delta\sigma) < 0$: time interval $\gamma_\beta(t) < 0$ and $\Gamma_\beta(t) \leq -\frac{1}{2} x_+(\beta)$

  sufficient condition

essential non-markovianity

Unital case $\Gamma_0(t) \simeq 0.056$

[Gherardini et al, PRR 2 033250, (2020)]

CONCLUSIONS

- s.e.p and fluctuation relation for non-unital quantum maps
- Example of irreversibility mitigation in non-markovian, thermalizing dynamics

ArXiv: 2210.07866

& OUTLOOK

- Increase system size
- Non-invertible fixed points

CONCLUSIONS

- s.e.p and fluctuation relation for non-unital quantum maps

- Example of irreversibility mitigation in non-markovian, thermalizing dynamics

ArXiv: 2210.07866

& OUTLOOK

- Increase system size

- Non-invertible fixed points

Thank you!